Title: Visual Math

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🔊[0:00]

Thank you, good afternoon.

Thank you for coming.

I said that my language is also not English like yours, so don't worry.

I will speak with mistakes and I hope you will speak doesn't matter maybe your English is better than mine, so please don't hesitate.

Okay?

I believe that when my child come from school, back from school, I would ask him, how did you do at school?
I actually want to know did he or she ask a good question, What question you ask your teacher, I don't want to hear how you solve your homework or the exercise, I want to know what kind of question you had to your teacher today.

I think it's more important.

So I would be very happy, if I'll get the question from you because this is very important part of my philosophy idea in education.

And that's the word inquiry asking questions and for many many years, over thirty years, three-zero, I do soft words and whole reason to use the soft word is to make students ask questions.

And ask interesting questions, not questions that have immediate answer, but questions that need some thinking.

So that's most of that reasons why I do technology.

🔊[2:27]

Let's start.

You will tell me when to stop.

And we'll see.

I understand that you have the notes.

I decide to skip some from time to time and leave it for you to raise questions to me.

Okay? So any time that you have question, you raise your hand and if I can, I'll stop and ask you or I'll do it shorter like thirty minutes and then we will have time to discussion.

Because I really want to learn from you.

Okay.

So the challenge was to do different school mathematics.

Different school math- Usually when people think about different education they do it in science, they do it in language, they don't do too much inquiry and rethinking in mathematics because mathematics from most people is practice.

So we wanted to think about it.

Can it be different? I want to better understand what these things what you are using and I am using and whole world is using smart classroom.

What do we mean by smart classroom?

Because actually I don't want to smart worlds.

I want smart heads.

And I want to do it in the secondary school because usually and especially in mathematics we do it in elementary school.

And then secondary school and high school oh- you need to be serious.

So in elementary school, you can play with technology but not in high school.

And I want to do it different.

Just as you I think, just that's what I heard, I want to creative learners, and I want it classroom, student center classroom.

But how do you do it? When there are thirty students and one teacher.

And how can technology help me with these, this is the other question, and I want everything what we do to be developed with teachers with school, not in the laboratory at the university.

I'm at the university and then going bring it to the school but to do it with teachers

because we are talking about changing norms.

Teaching will be something different.

Text book will be something different.

These are all norms that we need to think how to deal with.

It's not just bringing the computer to the classroom.

So that's the rational, I know that, oh maybe I should ask, How many of you are interested in mathematics? It doesn't matter if you are major in mathematics, just if you are interested in mathematics.

Do I have any? Safe can help the others? Nobody.

🔊[5:36]

One.

Okay.

So I have now the hard work to convince you mathematics for high school is something interesting.

Okay, there is a quite famous European mathematician.

He already died.

by the name of (Pál Lakatos).

And he is not mathematic educator, Mathematician.

And he is one of the good example what does it mean inquiry in mathematics.

It means you have conjecture.

Do you know what conjecture is? Could you translate? Conjecture is like a question, but educative question.

Okay, So everything, every mathematics starts with conjecture.

And then you do experiment, you think, you find counter example, you maybe find proof and then examine if the proof is correct and so on.

That's how mathematicians are doing mathematics.

This is not how all of us we are started in school.

Correct? I mean I don't know.

That's not how I studied in school.

How did you do in school mathematics? usually not by making conjectures.

So, That is a big change.

The other vision that they have about inquiry learning is not connected to mathematics, it's more general, so maybe, it will be, more familiar to you.

I want to think about curriculum, like I think about an exhibition in museum.

Because when you go into a museum, somebody made this exhibition, somebody made decision why one art is, one piece of art is on the wall next to the other.

Why there is these pieces in the hall.

It's not just random, I want but, still I can go in the room and any order that I want, and look hear and come back and get guidance if I want, you know I can get real human guide or I can get now ubiquitous guide in the museum, so whatever it's like a classroom.

You can do it with the technology, you can do it with human but, I want to allow curriculum, formal curriculum.

I am not talking about infromal afternoon class.

I'm talking about formal school government curriculum.

I will tell you in a minute.

That's we'll look more like museum than classroom of mathematics.

And it will have implication of how digital book will look like.

Because with a paper book, it will be more difficult to do it.

🔊**[8:55]**

So you see that technology comes only when I need it.

Not because I decided that I had to use technology.

Everything comes because of the philosophy of what I believe.

Now, in the classroom, it looks like this.

I have some videos but we'll keep it for later if we have time.

There is a teacher.

You see what I'm doing is? Twenty years old, this thing.

It's not everything is new technology.

It started, for example, this is a classroom from 91, 21 years ago.

This is a software on phones from two years ago that we were working in the class.

So it's different.

But there is a teacher in the class.

It looks like a classroom, at least in my view.

Sometimes there are computers; sometimes there are personal equipments in the pocket.

And there is a blackboard, not smart board yet, although it needs where in my country there are classrooms with smart boards.

That's not a problem.

But the issue is not how the technology looks like, but what do you do with it.

So, I will raise these questions today and I will try to do all of these.

Let's just keep it.

Let's first look what I call "learning with understanding".

This is mathematics.

This is not in your note, I added it.

It's not in your note.

Because I thought I must've explained what do I mean "learning with understanding".

Because this is the most difficult thing to do.

In mathematics, usually what we do is we do manipulation, calculation, solving equation, simplifying expressions, doing derivative and integrals, computing volumes in geometry, you do it, right? As a student.

All that is here.

But this is only one thing in understanding.

And maybe not the most important thing.

I think it's not important thing.

I think it's important to know how to do all these.

But it's not important if you only do it, only this one.

You need to know who to do modeling.

Modeling is looking outside of the mathematics and know how to write mathematical model, that's something which is considered much harder.

We usually don't do it.

We do it very little.

Maybe the most important thing is reasoning.

You need to argue.

You need to know to explain.

That's to explain to yourself, to explain to somebody else.

🔊[12:12]

Why you're doing it? And in mathematics, there are special ways of explaining including proof.

But proof is not only way of explaining.

Proof is one thing which is unique to mathematics.

And there are different ways to talking mathematics.

It's not only alphanumerical letters like variables.

In mathematics, you can talk with graphs, with pictures.

That another place where technology comes very well.

You should talk with your body, with your senses and I would like to show you some examples.

So, four of these need to be there? for understanding.

Usually and that's why mathematics is a problem.

And that's why most students in the world, it's not Korea, it's the whole world, dislike mathematics.

How many students, especially how many girls, are going to do their PhD in mathematics or in physics? And the whole world is talking about that we need more and more.

When you ask them why, most of them say, "because okay I'm good at mathematics, I can be good at mathematics, no problem, but it's not creative, it's boring, I am a creative woman, I don't want to be bothered with mathematics".

I'm trying to change the image.

And I think technology helps me to do this.

Okay, I'm going a little bit deeper into mathematics now.

So I don't know if there is only one person with me.

I may skip it.

I think I will skip it.

Okay.

If you want to know more about the mathematics, I will go back because I'm going to talk today about algebra.

I assume each of you knows algebra, very simply algebra, school algebra, not anything that you need to be major in mathematics.

But, these are my examples.

That's what I know the best.

I will talk about technology but the examples will be from algebra, okay? And if you ask me how it can be generalized later on, I will try to answer.

Okay.

🔊[15:00]

So, the technology that I'm talking about is three actually generations of development.

As I said, it takes more than twenty years to develop it, a lot of research.

It started with PC technology in the 80s and 90s.

It continued with digital textbook.

I started to do it ten years ago on the Web.

And it continued in the last years with mobile phones and mobile learning.

That's the technology.

Before you came, I went to see your future classroom.

I'm not there.

I mean it's not any problem to do.

I mean you need money, you need ideas.

But I will talk more about the ideas that should be there, not about the technology so much.

Designing technology-based inquiry curriculum

🔊[16:25]

So first of all, what does it mean to design technology for six-year mathematics curriculum? In Israel, students go to school at age of 6 to first grade elementary.

They learn 6 years in elementary school.

On the 7th grade, they go to secondary school, 3 years middle school and 3 years high school.

So altogether 12 years.

Okay? I'm talking about starting at the 7th grade.

I'm talking about secondary - middle and high school.

But most of my examples will be simple one from the beginning of the 7th grade.

So please don't worry.

Okay? But if you think about developing a curriculum for 6 years, what kind of software do you need? I know in mathematics, I just want each of you to think about it.

It's a big task.

So in mathematics, I said I have to have software in geometry that will allow me to do construction, write, get feedback, do all kind of dragging.

You keep yourself markers.

And anything that you want to see, I have the software here, later on I could show.

🔊[17:59]

Okay?

If you want something.

So geometry we teach I need a big microware for geometry.

I'm talking about the beginning of algebra students in the elementary school talk about the numbers, when they come to secondary school they start to talk bridge letters.

I want them to be able to talk with letters naturally.

So instead of starting with expression and equation, they could start with different patterns of number and just describe them with software in an almost natural way, just write and get feedback because that's why I need the software here of counter examples, they remember the conjecture the counter example, the reasoning that that's another software that they need.

What else? We need algebra, we need graphs, you probably in the high school learn also functions and graphs, correct? So how do you started? Remember that in my list of understanding, there was the modeling to do models of the world.

Functions and graphs are the bad thing to do models.

So I want one of this.

I want them to work with their body.

Maybe, your people told me that it will be nice to see this one.

So I can tell you one of this.

This is for the very beginning.

Young children.

They just drew…wait.

Where is it? I can't find it….

A…See…I don't want to go out.

Okay.

I will continue here.

And go back… here is the example of the software.

Just do with your hands, you draw.

With your body.

You write your signature for example.

And what the technology does for you it does graphs of your drawing and then let somebody else come and try to do the same signature they can see that they have a different graph.

Okay? Or they can do graphs with icons there is no algebra here, nothing.

Just visual.

This is something that you cannot do without technology.

Okay so what I'm looking for in…in technology is sorry.to go.

🔊[21:03]

To I'm looking for technology that can do things for understanding not for checking if you have right or wrong answer.

That's, that's easy.

Okay technology can also do this.

But this is not the important thing.

The important thing is to invent and that's what I'm very famous I must say the most famous thing of my work.

Is to do software that help you to do to think mathematically.

Okay, then you need books.

So we develop the books and there are many revisions, for example, now here are ready at least four revisions, in the beginning it was paper book just like exercise

then we add some expositions here, some explanations and the definitions.

Then ten years ago, the same thing revised as the digital book.

And today, there are different like digital books but also paper books like PDF you know, that the PDF files that I augmented with the software.

So that different revisions when the technology continue, we also revised there is a assessment.

It's very important question.

How do you assess when you teach differently.

When you learn with software, do you want to assess with software? Would you want them to think without software without technology in the test? Yes, I want them to be able to think without computers.

Of course depends when and about what.

So the assessment is big issue.

And there are research and you have the finding I think on the notes rights? It…It is printed.

There are statistical a.

Okay.

I will tell you a little bit about the system in Israel.

Very similar to Korea.

One ministry of education, very central, we are small country, we are 6 million the whole country, same here.

So it's easy to understand and easy for me to do.

So this, the visual math was supported by the ministry of education, it got approval that it was not that compulsory curriculum because even until now, and certainly ten years ago, we cannot assume that every child, every students, and any classroom will have a…a…personal technology.

🔊**[24:01]**

So it cannot be compulsory, however it's done along syllabus, a central syllabus, and that taking the same test with slightly variation that fit the way they learn but without technology.

The assessment is….

So there is a lot of both quantitative and qualitative research and whatever is interest to you I will come back to the later.

I want to go now to the second major development, after software.

After software is the books I already said little bit.

And I want to go into details about the digital book, it's open on the web a… there is a link to this if you just type visual math function web book or whatever.

My name was my link a…a…you will get it.

I would say it's not technology smart in my… in its view.

It's more than ten years ago.

It was developed you see here, ten years ago.

It's in algebra and because it's supported by the government it include the compulsory topics.

Linear and quadratic.

But look how it looks.

A… by the way, it's on the web but it's also could be stand alone on the PC.

It doesn't have to be on the web.

It includes big problem, exercise, homework, exercise for self assessment, a….you will see.

When you go, so, just seconds, there are two topics here could be more.

Think about the ideas.

Don't think about the specific mathematics, okay? There are two big topics.

Linear and quadratic, it's about two, three years of mathematics of algebra.

Maybe seventh, eighth, ninth grade maybe even more.

If I go a…one of the chapters, I see a circle.

Why? Remember the museum that I had in the beginning.

And I said that it will be easier to handle with digital book? That does not go linear? Now what is the mean that it does not go in linear? There is a teacher here.

🔊[26:56]

It's not that every student decide here to pick something and do.

No, it's for the teacher.

The teacher is like the curator.

The teacher can decide where to start because there are different ways to learn mathematics with understanding maybe you want more manipulating, you will go to expression to equation, to solve it, if you want more modeling you have these chapters which are in different color if you can see, or anything you can decide.

So, I will pick one, I mean you as a teacher, OK? I am talking now about teachers taking the law of designing their own course with given textbook, rather than going page by page.

Like in a museum, there are too much here, I mean, no teacher will be able to do all of this, It is not necessary.

Necessary…It is too much.

So the teacher has to make decisions and we try to train teachers to make this decision.

Once you go in all the pages looked same any topic that you will choose will look about the same the reason definition, or beginning of the topic, there is big task written essay.

How many wrote mathematical essay here in school, none? I also didn't, but our students could do this.

It is like, again, mathematics is something else I mean, mathematicians write papers and send it to journal to write books right? So how come we never ask students to write anything in mathematics? So there are big problems like this but the also small problems I will show you in a minute.

Now, anytime that you open the book you will see around them example.

Always here, for example, it is this time multiply by this, you get this.

But you see, It is completely visual no expression, infinite examples.

I think that this is the first capacity of digital textbook to show different examples.

In any topic not just in mathematics.

Because otherwise students in is very know in the research they don't do not accept it as example, they accept it as their only example, that's what on the paper.

That's why there is best use of different examples it could be difficult for the teacher because maybe 30 students, each student has different example, could be.

That's why it's more you need to be a better teacher to teach with real digital technology I think.

OK there are tools that help you to do anything you want.

You can decide how to trains, I will not go into the mathematics, everything is interactive here.

You have problems for example this one it's about rabbit cage, and it's dynamic geometry, sorry.

You can every diagram here every example is this is the modeling to make a model to that rabbit cage everything is dynamic here otherwise what is different between digital and paper.

So there are many problems here.

And there are exercises.

In the exercise, you need to give an answer.

There is problem here to solve, for example, do this and this, write something.

But once I need to put to write something, that it will go through the tool red points.

Okay? I need to know a lot of algebra so I will do it for you, let's see if I can do it.

I don't know, because there could be many answers.

But I don't need you see, there is no anybody who is telling me that there is mistake here but I know that there is a mistake right? How do I know? Because it was supposed to go through here(red points) and it doesn't.

So I see myself that I didn't complete the task.

I don't need somebody else to tell me if I am right or wrong.

This is my identitive education there is a teacher that could help me if I ask, but I want to create softer to create books that will be able to tell each person, each

answer if it is right or wrong if I want.

So that's why I have more room here, and I can improve it, so maybe now, I think I know what to do.

Ok? Now, this could also be an assessment valuation I think, but maybe than the computer will know that it took me to tries to arrive to the question, to the answer.

Here, when I do homework, I don't mind, so yes I want to do it fast but if I have a mistake I know that I have a mistake.

🔊[33:30]

So, this is the principle here or the exercise so it is a workbook and a textbook and the feedback, we call it mirror feedback.

Mirror…okay? Because it is like mirror for me it tells me itself.

Nobody has to tell me.

Ok.

So this is the digital book and I will go back to the… to here we talked about linear, yeah, we just saw this one, so there is also tools in the book all the tools that you need all the software tools.

There are hundreds of them.

If you are looking that, there are hundred interactive tools and diagrams.

So we talked about this, there are definitions, random examples, writing an essay, doing exploration task, doing exercise, having tools, and interactive diagrams.

I will not go into that because this require little bit more mathematics but it says that for example to teach the compulsory, algebra of this topic there are 3 ways to choose and the teacher has to choose or I, as the curriculum designer, could suggest the teacher what to choose but in a () digital environment there should be more choice otherwise I don't understand why do I want to have a digital book.

Just to say that I have a smart classroom or just to make it more modern? it is not enough for me so there is a lot of research that's how we looks in the classroom they sometimes were personal, sometimes they sit in small groups and that's different but it is possible I'll go quickly to this third generation than I will finish.

Is that ok? Of shall I finish now.

Designing ubiquitous learning environments.

Two more minutes, ok.

The mobile….

🔊[36:03]

Look.

The mobile… I think it is interesting because it's the first time that we will talk really about personal technology.

We talk about personal technology, PC (Personal Computer) for thirteen years.

But in school, it was never personal.

It was the school computer, right? To work with a very simple mobile phone so simple that now that are not working anymore.

If you would go to this site, you will be available to see it and also "YouTube" short moive(movies) that explain what it does.

It takes material.

Oh I am not connected to the….

Oh ok, never mind.

We will do it later.

I thought that we have WIFI.

You can do it yourself.

If you go to "YouTube" or Do you have this link in your notes? in the print-out? Oh, ok.

If you are interested in, just look and it shows it's nothing about the mathematics but it is about interactive classroom with a personal phone.

Aw. I took pilot very little maybe five and six of that hundreds that I have on the digital book hundreds tools and interactive diagram.

And Put them on a simple mobile phone.

On all NOKIA even before all Samsung, all LG it works many all phones.

Because it's really meant to be cheap and personal and I was interested to know what can we do beyond the computers, laptops and tablets.

I think there is something very special in its being cheap in your pocket and belongs to you and not to the school.

So, here, this is smart board classroom with the phones.

But what is mostly interesting is a communication because after all, phones were made for talking.

These days, we are hardly to talk with the message, right? We write short messages.

So, what the applications allow is to send short visual messages.

So, for example, here is a task we ask from the students to take the short video there, phone camera and then, to use one of the applications on the phone to do sketch and then to send it to their other students and to the teachers to see if the sketch really to describe what happens in the video.

🔊[39:10]

This is the very nice task from mathematics that we usually done to do.

It is very easy to do when you have mobile phone camera in your pocket.

So communication is something very interested in because that was something communication I mean from distance.

Communication inside the classroom I had but communication from the distance something else that I want to learn more about.

There is also system of clickers, do you use them? That the teacher can pause the question on the board and students can click so it's on your mobile phone, also.

If you look at the YouTube video you will see of this there is an animation that is describing it and the last one, it's back to textbook…look.

I think there is a still great value to paper, books.

Maybe I am too old.

But, I am doing always technology so first I think that there is a value to paper there are things that much easier to do on paper.

Reading text, reading expressions are easier to do on paper, than on personal computer or any tablets.

And also, there is 75% of the world.

That it will takes still many, many years until they will have I-pad or tablets in

available for them and talking about most of China, most of India, most of South America.

And I am really interested in how to technology could help people be more smart and more clever all of the world, all student and not just rich one.

And aw I am trying to so augmented textbook something that I started a few years ago.

Here is very all calculus book I had as a student.

I replace one of the diagram with the bar code.

The minutes that I point with the software application with mobile phone on the bar code I make this diagram interactive.

🔊[42:00]

So I actually have the capabilities of random dynamic examples that I show you on the interactive textbook with a paper textbook.

But so, this was the first experiment two to three years ago.

We are now designing something very new.

You need a little bit better phones paradigm.

And this is the augmented reality where you don't need even to bring the bar code because Printed bar code, is probably means that you have to print the books again with the bar code.

There is an option that you can add bar code on another page just to add.

But so, today's technology allow visual image, analysis allow to do without typing so the idea that we are working now which is the very neo-patterned.

All the last of the material are free.

These are patterns, inventions for the augmented textbook.

Aw~ we allows you to create a community of reader.

On a paper, textbooks, and that should be interesting because you will be available to scan the book and see in an augmented reality like I saw you in smart classroom.

You don't need any marker and any print in the book.

You will be available to see it here.

You will be available to leave the question to get the answers and so on.

And paper book, we still be called, so just to conclude maybe this would be easier for educational system for policymakers, for teachers to have the core remain stable on paper and have this augmentation with available technology.

Hm, alright! So smart education I have one your keywords, speaker yesterday in the conference.

Do I say, right? K… Korean institute of research and assessment, something? Yap, she was talking about smart education all the time and I also actually saw one of the on your site publication.

This is from your site in English about the smart classroom.

And what you hope to do and smart education.

This is from one of the news papers on your site.

So…ah I hope to think ….help to think a little bit about different ideas of what smart students can be and that will be smart education.

🔊[45:00]

I suggest you to do "Technology –Based but Not Technology Driven".

Bring your ideas and use technology to do.

My idea is to support conceptual and skill and inquiry just delete mathematics anywhere.

I suggest you would think evolutionary rather than revolutionary.

Like I did in 25 years, every time the new technology class I upgrade the things the idea……is the same idea…is idea of…contents ideas of what does that mean understanding and I think that you need to think about the long term change because you must know you are educator.

That education is not for quick changes like technology.

It's not working.

It takes the generation to do re-changing education.

25 years it is not long time education.

And 2years, technology is old.

So, that's why I think that we need not to design upon technology but to design upon idea with a school.

Thank you!!!!