Title: 시장설계이론1, 이산 자원의 분배 (4)

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- ✓ **Dictated: 강은경, 강성호, 김신희, 김종백, 신원대, 현소형**

[00:00]

Okay. So, let's go back. So, we have shown that the PS assignment is ordinaly efficient. Okay? Now, what about symmetry? It's symetric clearly. Why? Because if you take 2 agents with same ordinal

preferences, their rotarys are exactly the same clearly. Right? So, if you fix the expiration date, you know

we can see that their preferences.. If you take 2 agents with same ordinal preferences clearly,

their times each agent spent for eating each good is exactly the same.

So, you get symmetry automatically there. You get something more than symmetry which I didn't unfortunately say.

But you get in fact..

we get,in fact, envy freeness , what is envy freeness?

well, assignment is envy free , if any angent prefers the assignment he gets, the lottery he gets to the lottery is that all other agent get ok?

when we now, so, if each agent likes what he gets more than what everybody else gets then we say the assignment is envy free

but here, when he say lie, we have to be careful about in what sense he likes ok?

now, since the preferences are only defined in terms of ordinal rankings

so that means that we have to also define liking in terms of ordinal rankings

so when he say I like my lottery better than everybody else's lottery

what I mean is that I like my lottery more than other guy's lotteries in terms of first stochastic dominance dominance ok?

so another way is that if I , there is very strong sense right? So another way is that you know, it could be , we can possibly see, mean that

this two lottery may be incomparable

what I am saying is that that is not even the case

so if assignment P , random assignment P is envy free if every agent ok?

likes his assignment , so his rows ok? More than other people's assignment

another way is that all the other rows except for his ok?

in the sense of first stochastic dominance ok?

so give me any good the probably getting A or better ok?

ah is higher under his row than everybody's row

that's what we get in PS

I am not goint to show it , it is in the proofs in Bogomolnaia and Moulin

That is not the case however under RP ok?

so envy freeness define in this way is stronger than symmetry

so mechnism can be symmetric and yet need not be envy free in this sense

RP face to envy free in this strong sense

Because , first of all, it's very requirement in the sense that what we are talking about is that all the lows are , must be comparable in terms of first stochastic dominance

it's actually I am not , I am saying someone inprecisely but you know,

what I mean is that there is a row corresponding to I ok?

It must dominate ordinally all other rows ok?

accoding to his own preference ok?

and likewise for every other J , rows

that is satisfied under PS assignment but the same but this is not satisfied under RP assignment

so in terms of fairness, PS is even better than RP

the only defect if you will of PS is that it face to strategy proof

which I am going to show

here is an example to show. That, make that point

there are three goods, I mean two goods , A and B

and of course possible null good and there are , M copies each object ok? And here is more than two ok?

[05:00]

M copies each object

and there are three types of agents , there are many agents potentially

agent I likes A more than B , B more than having nothing

there are M nunber of agents with this preference according to each

A is the only acceptable good ok?

likewise there are M+1 agent who have this preference

now suppose all the agent , every agent report truthfully ok?

every agent report truthfully , then M+1 agents are going to be eating , M copies a good A

and M+1 agents here , this isn't here , start eating A , there are , how many of A is?   Mas ok?

but this many guys eating B how many of Bs? There are Mbs ok?

so this is the case where everybody , so what we are going to do is we are going to fix the preferences for all agents except for I

and I am going to vary I's preference report to make a point that it is best interest to lie

in this case, given that everybody else tells the truth

so what we did was the case where everybody including I tells the truth

so in this case, Ta equal Tb equal than M of M+1 ok?

because there are M number of good A , the M+1 guy is eating at units B

so therefore at time this equal to that , A disappears

and from then on , also B disappears so from then on you get nothing from here

these guys eat A , so the probability of each of this guy eating A is M of M+1

and the probablility of this guy eathing B is M of M+1

when everybody tells the truth

I tells the truth ok?

now, I lies ok?

and reports this way ok?

so then he starts eating B ok? How many are eating B?

M+1 guy aside from that guy M+2 people eating B

untill because there are more people eating B than A , B will run out first

when does it?

well it is sqare T because M divided by M+2 right?

now how many guys have been eating A? M guys have been eating A rihgt?

so A lasts longer clearly ok?

exactly when? We need to learn a little bit more to figure out right?

because upon to here M guys are eating A so how much of B , way has that they have eaten , M times this much ok?

from then on , how many people are eating? M+1 guy ok?

M+1 guy for how long , I don't know , we have to solve for this

and what we get at the end exactly this

[10:00]

I am not saying that delta is this , delta is different between this and that

so I am saying is that this is the delta

we solve for delta in this way and then once we do that , we can figure out this ok?

important point, however, is that this is greater than this , not suprising

because , now, if everybody were to eat, the M+guy have been eating then it will last this much

point here is upon to this time, only M people have been eating

so therefore this must be, the total time must be here longer ok?

because smaller number of people are eating here during this time

so what does that mean? In terms of I' chair of eating alternative goods upon to here he eats A no he eats B and during this period, he eats A ok?

if he's own us between A and B then I am saying this is better than that

why? Because the total duration here is longer . The probability of eating something is bigger here and here

probability of eating something is this here

probability of eating something is this right? This guy here ok?

Ta tilda is greater than here ok?

now imagine that this guy is totally indifferent between A and B which are better than that

imagine now that we give him cardinal preferences such that here is Von Neumann–Morgenstern utility value is Va and Vb strictly greater than that

but suppose that this is exactly this ok?

and you let epsilon goes to zero

if epsilon is sufficient , small enough you like this more than that

so in that sense here we see the failure of strategy proofness ok?

so the problem here is that it can , when we switch , when we manipulate this preferences to lie

he likes A more than B and yet he said he like B more than A

C is that it is not so bad because well by lying that way he speed up the eating of B

so B runs out quicker than otherwise and then after all he enjoy A ok?

now, imagine however that if M is sufficiently large ok?

then his ablility to change the eating speed that will very very limited ok?

so by switching , it is just one little speckle out of the universe

so he is not going to able to affect the eating speed of different goods

and therefore the expiration date we talked about would not depend on how we report ,just one guy out of huge number of object , huge number of agents

so we have to when we say , when we enlarge the economy , we enlarge both so, we keeping the types of objects constant ,say there are only two goods

that we increase the number of copies of each object

and increase number of agents hope different kinds of preferences the easiest way to expand economies by relicating in this sense

doubling each number of copies for of each good , doubling the number of agent of particular ordinal preferences if you doing that ok?

that will be easiest way to replicate although the result I am about to say is not going to depend on a particular expension enlargement technic like replication it holds it more generally

[15:00]

but the point that this , there are two messages
one is negative one namely that Ps is not a good mechanism in terms of stratege proofness it fails stratege proofness

there are cases where it benefits its patient agent to lie

the second part of the message is that it is not so bad you know it is large enough economy ok?

so that the existing example is meant to make both points right?

so first we show strategy proofness failure

sencond I will show you , as you can easily see it

as M goes to internertive it is the number of copies and increase number of ordinal types

then you can see that this converges to one ok, that converges to one as well ok?

and therefore in the end the agent , this agent I has no incentive to lie ok?

so even if this two are the same ok? If we are indifferenct that does not matter

of course he is not lying at all

if he is not indifferent he likes A more than B

now once you fix epsilon no matter how small that is

fix epsilon and then increase M , there come up from where who want to lie

so here when you do this kind of argument you have to be careful about the order of taking limits

so we fix epsilon and let M goes to infinity , that is what we do

the other way does not work right?

so in other words that fix M and let epsilon go to zero then of course at some point you lie right?

so we have to be careful about in what sense we are talking about

and here is the point that [?17:8 kosama and Nlyania] wait, which is to say that if we fix the cardinal preference types so cardinal preference tpyes from some finite set of numbers real numbers

you draw cardinal types form that ok?

that means that the finite is means that the gap between any two is limited , at boundary about zero

so you draw cardinal values of preferences Von Neumann morgan strategy value from some numbers

so there is minimul difference , minimul difference between any two is bounded away from zero

if you think about that , in other words that here you know the situation that you keep , you fix epsilon ok?

and then you let M goes to infinity in the following sense , in the sense that number of copies go to infinity number of I mean

you, ways number of copies , ways number of agent given preference types

in fact, I mean I said that this result for all generally then replication economy

in the sense that as long as the as, long as the number of copies for given object divided by maybe the largest , the entire population

so let's say , so imagine the following

let's say there are equal number of copies ok?

M copies of each object ok?

and then also there are , I mean even it works more generally I am giving you one version

M copies of agent and then there are let's say f(M) , so

a particular type there are

ok let's do this way, let's fix particular ordinal type ok?

P is just particular ordinal preference ok?divided by M has a limit

it could be , it could go to general but it converges to some number ok?

and replication economy is special case

in this case, it doesn't depend on M ok? So some fractions always ok?

[20:00]

so in that case, so this result was namely that if you fix the Von Neumann morgan strategy value , possible Von Neumann utility value is that you know one can take for a consuming particular good if you let the economy get large ok?

economy that is sufficiently large , we get strategy proofness back ok?

[student speaking]

yes, envy-freeness is always maintained along the way

so envy-freeness holds for any given economy

so the problem was strategy proofness because it was not satisfied for final economy

[student speaking]

sure that's very good point because envy-freeness and strategy proofness become same in some sense you get it very easy , easy argument there rihgt?

envy-freeness implies strategy proofness almost

well because you know, there may be agent that you may lie in a way that doesn't match up with anybody else's

so it is right to say strategy proofness implies envy-freeness , it is a little bit trick to care say evny-freeness implies strategy proofness

because there maybe preference type that you may , you could lie to

which is not the prefernce type of anybody right?

the point is that ,right

so what is saying it this ok?

so in the large economy, let's in the limit this economy that we are talking about ok?

I particular , I here is , we cannot say particular , we can sort of talk in terms of the preference tpye ok?

so think about an agent with particular preference types and now we index that by I ok?

mixing up rotation here unfortunately

because P here means the row of this random assignment ok?

you can sort of imagine still describing a random assgnment in this even large economy with a matrix like that finite matrix why?

because each row here now represents particular ordinal preference type , not an agent

because there are large number of agent you cannot possibly do that then

but there are finite because there are finite object types ok so finite object type , each available , there are each number of colons or copies

ok? Number of copies explod but the number of object types remain fix right?

so therefore there are finite object types and finite agent types

so we can solve this kind of in terms of types, that is what I am saying

now so I would like to do that

so let's say this is the particular row corresponding to ordinal particular preference tpye ok?

what does the strategy proofness imply?

strategy proofness implies that so I is now associated with particular not an agent in the X, index of particular ordinal preference type.

one out of , if there are N object types, N factorial ordinal preferences.

now, so this means that stratagic proofness means this,

now so this means that stretegy proofness means this ok?

I is ordinal preference type

now I can lie to any one of n factorial ordinal preference type right?

stretegy proofness means that this is true right?

I incentive to lie

but here again you have to be careful about what you mean by liking

[25:00]

again in the sense of first of the stochastic dominance ok?

D lottery that I get when I report my ordinal preference type truthfully

ordinally dominates a first of the stochastically dominates any other lottery that I would have gotten

by reporting some other preference type ok?

so this is true sp implies this ok?

in the finite economy if this were the case

that doesn't necessarilly imply envyfreeness ok?

because it could be that actually the other where uh this is actually easier envyfreeness does not imply stretegy proofness ok?

envyfreeness means this finite economy I get this and I like somebody else's trap lottery ok?

but that doesn't mean in that if I report this guy's ordinal preference ok?

I'll be worse off

so remember that I claim that the ps is endy free in the sense

but face to be stretegy proof

how can it be ? Ok?

ordinal's envy freeness implies that if yes I that every preference types represent in that economy

in this economy there are this is positive number of agent

agents we particular ordinal preference types

every types represented ok?

so envy freeness in that sense in that case means that

I don't want to envy everybody else I don't envy everybody else

or I don't envy D lottery of other agents with the different ordinal preference ok?

which that's mean however that very sound similar doesn't mean however

that we have stretegic proofness ok?

because if I even though this is the case if I actually

it could be that if I lie ok?

and report something else like here ok?

then that doesn't mean that I'll get this right?

because I'm going to change eating speed essentially

so that's exactly what' going on here

here in this case if you think about what happens on the first case

ok

I don't envy anybody else here ok?

becayse if I repoted here like this if I reported

this bar here for instance

gets exactly the same so there is no reason to envy ok?

how about that guy?

that guy's lottery means that he has this much share of B

and that much share of nothing ok?

so my lottery first of the stochastically dominate that lottery ok?

but remember that didn't mean however that we have strategic proofness

because when I lie all bets are off

this guy no longer that guy no longer in fact consume this much share

because I am affecting not only my lottery

I'm affecting other guys' lottery as well ok?

so therefore envy freeness doesn't coincide stretegic proofness

in the finite economy but I'm saying in this point out

in the large economy in the limit economy the other same

because when I change my report

I'm not going to affect the lotteries' consumes by different types ok?

in other words that if I were to lie ok?

that's exactly called lottery that people with that type currently enjoys

there is no distinction between the two ok?

if I have no incentive to lie therefore I'm not also envying

if I'm not envying envying that guy's lottery

[30:00]

that I don't also have incentive to lie in the large economy
becase my ability to influence other people's lottery is near ok?

but this point is easy to see at least ok?

it's very intuitive in the sense ok?

so you can think of the eating speed if you will as the price in a competitive market essentially

now a popular highly demanded object will be eaten very fast

because there are so many people demanding it ok?

so the good with the high eating speed is like a high price good ok?

now we know that competitive equilibrium becomes incentive comparable

if this result, famous result studying the incentive property sublarge market

suggest that incentive property gets very very good in the large market

because why is there a problem in the small market in terms of incentives

why because by changing preferences

I can manipulate price against especially monopoly market of course

you can change you know your behavior influence price that usually undermines incentives ok?

in the large market my ability to influece price

by changing my behavior is limited

the same thing is going on here ok?

eating speed A is independent on my report

that means that the expiration date will be independent of my report

and therefore I have no incentive

and B freeness implies that in fact stretegic proofness

so now that's sort of nice result

what does it suggest?

that mean because the lack of stretegic proofness was the only defect of the probabilicity of mechanism

so therefore in a situation like this

namely that economy's sufficient large will the finite sort of objects we'd lot of copies of an object, a lot of agents

then perhaps p s is not so bad

may event better that rp

because you then get the all 3 properties

impossibility doesn't hold

the impossibility means that you cannot get all 3

now you can get the large enough economy

but then what will be there sort of natural application of this?

what kind of setting do we expect to see this kind of features?

where is like a school choice?

you know there are finite schools a number of schools

each a large number of seats like hundreds of seats

so therefore that's kind of natural application of this

housing allocation also is a sort of natural

you can sort of have large number of different building types

each building has a number of suite same types of suite

so your preference is over I mean often you care about you know what floor you live in

you can sorf of divide in that way

I mean so maybe there are like you know maybe ten buildings and floor is what you care about right?

in that case there are 10 copies and object's type is floor maybe right?

but you can sort of think about many examples where these sort of framework

large economy framework is fairly well realistic ok?

so school's choice being probably best example

so as far as such an framwork

context is concerned maybe it's not so bad ok?

ps is something that we wish push for

I mean now why not use ps instead of p

ps is just a somebody's imagination but it should be used right?

because it's even it seems too bad to dominate p I mean it's better in any in that case

and then comes in sort of my paper

one of the two job market papers

and then I kind of sort about the implication of it

there are implication that they push they are pushing for

there is implication was that you know policy implication is that

we should use ps instead of rp

but I'll say wait a minute

how can you say that?

without knowing what happens to rp in the large economy

we just we know that ps is fairly nice in the large economy

[35:00]

but we don't know what happens to rp ok?
and then comes inside came the inside suggest that in fact

It's already pretty much to say that

because it turns out that in the large economy

they become indentical they become indistinguishable

there are exact same mechanism ok?

now I'm not going to that their argument is fairly wrong and wrong actually not wrong fairly wrong

and involves I'm not going to say the details

but there is a set of interesting more creative part of the argument

that is sort of the was a break through in terms of this policy

I'm going to tell you a little bit about that

now you will look to talk about to be able to actually show that these two identical

as the economy grows in this sense ok?

you have to be albe to compare these two

what happens in these two to model to assignment

you have to be able to compare them in a way that can easily see how they differ how they similar and so on with each other

well now we know one thing that is very nice of appear as which is that you can talk about you can understand what happens in terms of radom assignment

all you need to know is the collection of expiration date T b T a T c and so on ok?

once you have those you know everything about

I mean of course use to know their preferences that can dominate lotteries

because but you are talking about the same model right?

so preferences are fixed ok?

now can we kind of represent what happens on the rp ?

in a similar way ok?

sort of graphically visually similar way

now but it's not up here

think about rps that we order the agents in a random way right?

and then run dictatorship serial dictatorship

but how do you sort of how do you compare them?

here is a way

in sor of doing that you can find you can define rp in an equibriliant way

so I'll do that

you draw lottery number between zero and one ok?

and randomly draw the number for each agent ok?

and then the guy with the smallest lottery number is ordered the first ok?

if you happen to have two the same lottery number, then you will break a tie enough through way

you don't have to worry about that

because this is a real number between zero and one

the chance of the more than one getting the same lottery number is zero ok?

so therefore you do that

you get the lottery number and what's the meaning of the lottery number that's the ordering

that's one way to think about it

but another way to think about it more closer to here is the following

so you get number let's say point 3

there is essential computer terminal here

where we have to come here and log on and then make a choice ok?

you'll get, draw lottery number point 3

so this is time once from zero to one

suppose you got it ok?

then you to arrive at point 3

at time point 3 and then make a choice at that point

of course when you do that of course time stand still ok?

so that's what you do

then it's going to preserve the order order in terms of lottery numbers right?

now so then how do you do it then from then on?

so what is something that is comparable to the expiration date ok?

well here is the following

what's the largest lottery number that you can get and still

be able to consume a particular good ok?

and that's exactly the expression day under rp .

largest lottery number that allow you still to consume a given good ok?

now if this doesn't feel very familiar to you

think of this expression day as a cutoff

we called, used to call couple line

think of the numbers as the test scores

[40:00]

and each object is like each department in each university
and this cut off score there's the cut line

the way we use normally cut right?

but the cut line what cost the score is not runs opposite smaller score is better here right?

so you have to have a cutoff means that you have to have a number below the number you know

to be able to get into their department ok?

so if you are, have this through comparative exam environment

the notion of cutoff is so realistic right?

so you know that if their cutoff for department

a B is point 5 then you'd better get a score better than number below equal to point 5

so how do you determine the cutoff

it depends on preferences

if a given school B or given object is very popular

there will be a lot of people like to consume it ok?

their consume in this sequence of arivals

and then at some point, your arrival time is not good enough to get it

because this is all gone ok?

so last guy who gets it determiones cutoff essentially right?

that's how you do it.

now if you define therefore the cutoff point in terms of this lottery numbers under rp ok?

so there is cutoff one for like this

what we have shown is this

what we have shown is that this is the random number in rp of course

because you get a random lottery number right?

but as you as the economy increases ok?

this guy converges to here that guy converges to here, and this guy convers to here

in this probability distribution ok?

the probability that this guy

think about this max ok?

ok um ok so

oh acutually this is not the probability this is great then epsilon goes to zeeo

the largest gap of all largest gap between the cutoff here expression day here and expression day here goes to zero

or in the probability of the gap is strictly positive goes to zero

ok so still if you think from the perspective each aagent ok?

what happens here and what happens here is kind of not very comparable from each other

because here you are enjoying you are consuming every good every point of time ok?

but here that's not the case you come in at some point and them make a choice at that point

it's like quite comparable what's going on

if you sort of now switch perspective from that of agent to that of object then they are comparable

you can ask the question about how long a given good is last here

and how long a given good is last here ok?

what's a speed at each the given good is consumed they are comparable ok?

here each good is consumed by every agent each good is consumed by one agent at a time ok?

but they are comparable

we can show that the consumption laid of each good is the same as here and and here ok?

[45:00]

because each determines here in terms of the eating rate

here determine by the arrival rate ok?

the arrival rate here is the same as the eating rate here

because essentially the arrival rate is determined by the uniform random assignment

random lottery of the you know uniform random lottery

and here your eating speed is uniformed right?

so therefore in the limit they are exactly the same

that's the sort of simple way of telling you about the intuition behind this result ok?

and if you think about [?45:59] Q for replica changing of rotation here and for alternative replica forward right way.

so here what it mean by then let's go back to this example very first example that we had

Q is right Q used the term Q

there is the one copy of each object ok?

you can sort of increase the cloning rate

I mean so Q could be two in each case

there are two copies of A two copies of B and then there are four agents of this type there are four agents of that type ok?

that's the doubling you can imagine tripling and quadrupling and so and so forth

now this case we know that under rp.

this is that you can consider of this measure the misallocation probability

that's the probability that each agent gets the last preferred of the two object

there is one twelves ok?

you measure that 4Q equal one

you measure that 4Q equal two and so and so forth

so misallocation probability you want that to be as small as possible right?

the draw the misallocation probability in the beginning

it used to be a very small number here

but you know it has two one twelves

when Q is equal to one it has one twelves ok?

It goes down to zero.

(Student questioning)

That's true for application economy, but I am saying that this result is true for more generally.

So only require is that … oh, so it doesn't matter.

It doesn't matter how … each object time is available that are Q copies of each object type.

It doesn't have to be exactly Q, they can be all different in fact.

Some become bigger much faster than the others.

Virtulally no assumption needed in fact.

So the number of ordinal preference type also doesn't matter.

All we require is that … so let's say measure ...

This is sort of the number of people with particular ordinal type P in replication Q.

All is that converges to some number in R plus as Q is infinitive.

It could be that it can converge to zero, for instance as large economy that preference we are talking about.

It is very small fraction of population having that preference relatively.

And in fact, I mean, if everybody, if something blows up for instance, that's also fine.

That simply means that … well it's a little bit difficult to talk about when something blows up.

[50:00]

But we are talking in relative terms.

So relative size of agents with particular ordinal preference type can vary in different ways as long as there is a limit where define.

Let me just quickly mention one thing.

We haven't so far, we have not talked about at all cardinal mechanism.

Everything that we talked about is ordinal.

There are two mechanisms, RP and PS, the main important scene that ...

Given that they are the same in the limit, the large economy, there is no reason to favor one over the other.

Perhaps RP is easier to use.

Because there is a business of implementing it, also I haven't yet talked about.

With the reason needed for PS to be used.

We have to be able to implement it as well.

There is more computing needed.

There are other issues also that I haven't mentioned.

For instance, if you think about how the Columbia university, for instance, assigns dormitories for undergraduate, maybe even for graduate students.

They do exactly this version that I used as a way of representing RP in fact.

Interesting enough, they do it, meaning that they are randomly assign the time.

And then you get a time, you are allowed to log on your computer at that point.

You are clear to make a choice.

I mean you can log on anytime, but you cannot make a choice before that time.

Once that time arrives, you can make a choice.

That's exactly what Columbia university uses.

So I was surprised because initially why would you do that, why not simply have them express preferences of housing types and then you do exactly …

This mechanism takes time.

But then there are other sort of benefits of doing this way, the way they do it.

Suppose you want to share your housing suite with somebody else, your friends.

There is group preferences sometimes.

In that case, what they allowed to do is that you can apply as a group.

Then you get the same time, and then you make a choice among possible different suites that are available.

At some point, there are no suite that can accommodate all of you and then maybe there is no suite that you like.

Then at that point, you can split up and make individual choices as well.

So this is a kind of flexibility you can use with RP but probably very difficult with PS.

So I guess the implication is that, you know, we know also we have a better handle of what happens on the RP than on the PS in the fini economy.

Because PS is strategy proof, so we can expect them to tell the truth, and we know what happens there.

We know actually that the we get ex post efficiency so on and so forth.

With PS, we don't know what happens in equilibrium.

The analysis is not an equilibrium analysis.

The ordinal efficiency is relative to stated preferences.

We don't know, I mean, how they will report, so we don't know what will happen.

From that perspective as well, RP maybe better.

So reversing the application in some sense.

So those are ordinal mechanisms, it is famous cardinal mechanism which is very creative as well.

Here is what happens.

House allocation problem and objects and agents.

Now here, the idea is to create the imaginary market.

Imaginary in the following sense.

So there is market for each goods some sense, one market for each object.

[55:00]

And each agent is given budget, some budget to spend.

Now, but you don't use money, you use like tokens.

So there is no value outside consuming, other than consuming … whatever they want to consume here.

So let's say each agent is given hundred points to spend.

It's dividable money.

And then you want competitive markets.

So what you do it as an each individual, there is a unit share price for each goods.

Per unit probability, good A causes 10 tokens, good B causes 100, maybe 20 tokens and so on.

That's a price for each goods per share.

So that means that if buy delta share of good A, this is what you pay and so on.

So this is your expenditure.

Your expenditure cannot exceed the budget you get which is let's say some M.

Everybody is given the same budget in terms of this artificial currency, maybe 100 tokens.

And then what you do, and then you announce the price per each goods.

Remember this is price per share.

So you can buy a fraction or share of each goods.

And then what you do is that you demand share of each goods.

That's your demand.

And supplies where define, if demand exceeds then the price goes up and so on.

It can them compete the equilibrium price, competitive prices.

Of the goods and market cleaning prices of the goods, and competitive equilibrium associated with that prices.

And you are done, what you will get at the end?

Each individual gets share of each goods may some to one.

So you get at the end a random assignment.

That's exactly the idea.

Of course you don't have to in fact have them participate in this competitive market

All you need is to have them report their cardinal values Von Neumann-Morgenstern utility values of alternative goods.

And then you can run a computer, pretending that in fact they are actually playing this market game.

So it's a matter of finding a fix point which actually may happen so easy.

But in theory, that's possible at least.

And then you can compute competitive equilibrium, and competitive equilibrium consists of lottery for each agents

So you get random assignment.

So what can we say in terms of the welfare implication?

Well, we can off-the-shelf, we can borrow from general equilibrium.

Because once the equilibrium exists, so all we need to show is equilibrium exists.

This is sort of nonstandard economy, so it's not easy.

So there is ... a local nonsatiability is not satisfied

If you get one share of an ideal good, that's your bliss point, so insatiable.

So you have to work a little bit to get existence which dated.

So there exists competitive equilibrium.

Now we have to also establish the force welfare theorem, in fact that turns out to be fairly easy.

So competitive equilibrium is pareto efficient.

So pareto efficient in what sense in terms of expected utilities.

So it's in the cardinal sense, not in the ordinal sense.

So the notion of pareto efficiency is different.

But it's in the ex ante sense, that's important thing.

They are talking about lotteries here.

So everybody's expected utilities cannot be uniformly improved upon by an alternative random assignment.
In some sense, they are using two well known result, the general equilibrium result, which require them to do some work, of course.

But the idea is kind of a very old idea, creating market where there is no market.

In fact, there is no market in any sense.

You cannot use money, but in some sense, what they are using is the shares of probabilities as currency essentially.

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That's kind of the sense.
But they use the artificial currency to create market, but in some sense, you don't have to have market.

I mean all you do is to have them report their preferences and then you can compute for the market outcome.

So market here is a way of coming up with a nice, desirable outcome in the welfare sense.

Second result that we need which we do here, and also we need it in the PS case is actually another result which is needed to implement random assignment.

Everything so far is in terms of random assignment.

So we have not taken an issue with whether or not a given random assignment is implementable, but we have to actually worry a little bit about it.

In other words, if you give me any arbitrary bistochastic matrix P, so row sums to 1, column sums to 1.

Or any substochastic matrix where row sums to 1, the column sums to a number less than Y=1.

Now once we have that, is it feasible, meaning that can we implement it?

The question here arises because when you given random assignment, this is like a marginal distribution, it's not a joint distribution.

In what sense is it a joint distribution?

When you talk about a lottery of assignment what we are talking about …

So there are some deterministic assignment.

And then you randomize over this deterministic assignments.

Random assignment is not exactly that.

It's a different object, so different sort of animal here, shows really what lottery each person gets.

In what sense are this lottery is that different people are getting sort of competitive with each other.

For instance, to illustrate the problems, suppose this is the lottery, this random assignment.

How would you sort of execute this? How would you implement this?

Well, you cannot simply do the following, you know, flip a coin independently.

You can't independently flip a coin here.

If you do that, then with positive probability, he ends up with two goods.

Or with positive probability, he ends up with no goods.

So problem here is that even though the objects that we express here are marginal distributions.

They have to be all realized, randomly theory needs to be realized in a way that satisfies capacity constraints which must hold ex post.

In other words that each row sums to 1, each column sums to 1 in this case.

But more than that, the lottery must be such that ex post in any assignment in the support of assignment.

Each person must get exactly one object, and each object must be assigned to exactly one agent.

How can you do that?

So the question basically boy sounds the following question.

If you give me any bistochastic matrix or substochastic matrix at the result hold in both cases in fact, let's just focus on bistochastic.

Can it be expressed as a convex combination of bistochastic matrixes, each of which has an either zero one entry.

What am I talking about, a bistochastic matrix whose entry is either zero or one is in fact a deterministic assignment.

We still require … this is any particular b? materix whose entry is zero or one. So no fraction assignment.

[65:00]

We still require the same requirement which is that sum must to 1, row sum to 1, column sum to 1.

So this is the legitimate, feasible deterministic assignment.

For any random assignment, if one can express that random assignment as a convex combination over such bistochastic matrixes.

This is called in fact permutation matrix, so just permuting agent.

Then that's enough. Why? Because then just rid of the coefficient.

With this convex combination, so the coefficient is value between zero and one, and they sum to 1.

That's the lottery that we are talking about.

As long as a given random assignment can be expressed, as convex combination of a deterministic assignments, then you can implement it.

If not, we cannot.

That's the implementation requirement.

The question then is that if the mechanism produces some arbitrary random assignment, satisfying bistochastic condition.

Can we expect it to be expressed as a convex combination of permutation matrixes?

And the answer turns out to be yes.

Any bistochastic matrix can be expressed as a convex combination of permutation matrixes.

This is a result due to Volkov, Von Neumann, there are two people, Volkov and Von Neumann.

So what's going on, I mean, and then [?1:07:13 Hillan Jack] actually come up with an algorithm which is very intuitive as a cycle finding algorithm.

I am done here, so let me just do one thing.

In a situation like that, what you do, look for any arbitrary fraction assignment.

Pick one any entry out of them.

It must be that in the support of the assignment that you choose with some probability.

There must be at least one deterministic assignment where agent one gets this good with probability one.

There must be in the support as assignment where a guy one gets good A with probability one.

Whenever that's the case, he cannot get anything else.

So that means that, let me just go back, he cannot get that good.

If he doesn't get good, in that assignment, who is the only other guy who can get the good? That guy.

So this is the case. That means that number guy 2 must get good B.

If he get good B, he cannot get good C.

Then C must be assigned to number 3.

This is not an algorithm.

This is one deterministic assignment, another one could be that he gets second good in which case he cannot get first.

Who else can get the first? The last guy.

So therefore in that case you get that.

Turns out that this must be 0.5 … This is very simple example, so it's very easy.

But in general, you can still constructively come up with a way.

There is a paper that I have recently done with Milgram and Eric Budish and Fuhito Kojima which generalize this Volkov Von Neumann theorem.

I mean some of you who attended the presentation that I gave couple of years ago might vaguely remember this.

I am not going to do the remaining part of it which is about that issue, how to generalize Volkov Von Neumann.

It building additional mathematical foundations and also it's kind of more technical in nature, so let me not do that.

On Fridays lecture, I am going to do school choice, so there is another set of lecture notes. You must already have.