Title: 시장설계이론1, 이산 자원의 분배 (2)

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The problem that will be in dealing with difference from standard problem in one important way which is that the object that are assigned indivisable.

You cannot sort of divide a given object into a million different pieces and then assign them some nunber of pieces to one agent and some number of pieces to another agent.

The way niche that this non-indivisibility in indivisaility is that problematic is because with indivisibility do not have natural currency essencially.

So in the standard problem, the way to solve, the way to generate an assignment that is desirable in the sense of pareto efficiency and so on.

You use standard market, meaning that you know, you have agent before object if you will.

In other word that the price for an object that is overly demand will go up, until there is only, until the number of,

until the demand equal essencially supply.

So essencially you are asking that demanding agent to outpay other people.

To have the claim that object, and that's clearness very natural nice desirable sort of welfare implication,

Because if you, that sort of makes the claim credible, if somebody says you know, I must own, get this object

because my social value of owning exceed the other's social value of owning.

Now, how can we make claim credible?

If you are false to pay as much as I willing to pay, as much as you claim that is worse for you,

then clearly this has very natural, there is no incentive to lie.

Money does a lot of good thing in terms of fixing incentive, therefore to generate the outcome, that is desirable.

The market is actually build on this idea of making incentives credible that require to pay as much as the willing to pay for objects.

Now with indivisibility, you cannot do that.

Because, if they have no money, but if their goods were divisable, then you can sort of use one particular goods

as sort of numeral and use it as currency.

But that is not possible in our environment , so you should solidly rely on the communication and they are cheap to all recently

Everybody says that something that is desirable, they say they might be higher than others

And it is very difficult that therefore elicit the true preference when you are limited in terms of the kind of incentive

you can provide, using monetary transfer.

That's one remark in the difficulty that I elute will manifest itself when we talk about you know, random assignment.

The other thing that I didn't mention is that we show that these three actions charaterized TTC.

It is also quite easy that TTC satisfy individual rationality.

Because always point to something, that's better than your assignment.

If your endowment happens with a best in the remaing set of object, of course, you point to what you own, which is allowed.

But you never point to something that is worse than your endowment.

So therefore, individual rationality is satisfied in TTC.

By TTC is much interesting is that in fact, this seems three things can actually characterize

There is no other mechanism advantage in this satisfying three different, these three actions.

So here, what we are talking about is hybrid case.

Hybrid case in the sense that some houses are owned, but there are houses not owned by anybody.

So some houses are privatly owned by agents but there are other houses that are owned by nobody, it's kind of publically owned.

So this is a set of mixture of house allocation and housing market problem.

What you do , while you kind of mix up two mechanism that you already know , serial dictatorship and TTC

How do you do that?

First of all, come put with some serial order of agents, all the agents.

Okay? And then, so there are 2 ways of it. So, we can do the following.

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We first think about all the agents or own houses. Okay? Take them out. Remove them.

And then you can put the serial orders on the remaining agents. Let's say there are 5 agents, 1, 2, 3, 4, 5

And 5 objects A, B, C, D, E. And 1 and 2 own A, B, C and 3, 4, 5 do not own anything. Okay?

And therefore C, D, E are publicaly own, socially own, but not privately own in the begining. Okay?

Now, we can put the serial order among 3 of them, and say that this is the first guy, this is the second guy, and that's the third guy.

Okay? So, and then we can do the following. For each object, we can put the..

So, who has high.. We can sort of determine the priorities for given object.

So, the highest priority goes to its owner if the house is owned by somebody in this case 1.

And the next own.. These are the priority order. Okay? Because 1 has the highest priority A

because 1 owns A. Okay? The other priorities are filled by non-owners. Okay? B is owned by 2.

So, same thing. C, D are owned by nobody. Because alternatively you can include 1 and 2 in the serial order.

And then, we include on the same. It doesn't make any differences. And then we run the TTC. Okay?

Treating that these are the kind of owners. Okay? [?06:51] here is that multiple houses can be owned by the same guy in this case.

Okay? That's fine. Okay? So, how will you do it? So, think of C as being owned by 3 essentially. Okay?

So, how do we do it? We didn't talk about preferences yet. But let's say that 1's preference is..

Let's say C, D, A, B, E. 2's preference is.. let's say D, B, A, C, E. 3's preference is C, D, A, B, E.

4's preference is E, C, D, B, A. 5's preference is A, B, C, D, E. Okay?

I just come up with the preferences in the arbitary way. So, 1 points to C. Okay?

And so let's do the other version. Okay? So, 1 points to C. C is owned by 3. Okay?

Now, 3 points to what? C himself. Okay? And 2 points to D. D is owned by 3. Right?

So, 2 also points to 3. Okay? And 3.. And, we will lead it 1, 2, 3. And D is also owned by 3. Right?

So, this is clear. Okay? This is all we have. It's one cycle. 3 is pointing to himself.

So, 3 and C.. 3 is assigned to C. Okay? And then we remove 3 and C. Okay?

What do we do? Like that. Okay? And the other guys move up in the ranking. Okay? And of course we have to remove C as well.

C is gone. Okay? Now, here, eliminate C here. Okay? So, 1 points to D. D is owned by now 4. Okay?

4 points to himself. Now, 3, E of course. Okay? To be actually in this case is useful to

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do the pointing through object because there could be multiple objects. So, 1 points to D. D is owned by 4. Right?

And 4 points to E. And E points to 4. Right? And 2 is also pointing to D. The rest is the same.

So, we did 1, 2, 3, 4. And 5 points to A. A is owned by 1. Okay? So, there is one cycle here. This is also self cycle.

4 is assigned to E. Okay? We get rid of E. And then get rid of 4. Okay? And then these 5 guys move up. Okay?

So, 1 points to D. D points to 5. 5 points to A. A points to 1. Okay? So, here is another cycle in this case where

1 is assigned to D. And 5 is assigned to A. Okay? And the we have left with one guy. And he will be assigned to whatever is left.

Okay? So, this is how it works. This is sort of mixing two mechanisms essentially. And remember here [?12:13] is that

we are running essentially TTC. But using the serial order as a way of assigning ownership. Okay?

And this method will be further used in the school choice algorithm that we will be talking about next time.

There is another way to this similarly one. Now, the same thing can be seen as used as special form of a serial dictatorship.

Now, this is a sort of TTC way of looking at the hybrid mechanism that is a serial dictatorship way of looking at

the same the hybrid mechanism. Okay? So, let's remember that C, D, E.. Okay.

So, we can think of the serial order which starts with 3. Right? Now, so, start with 3.

And what 3 likes the most? I erased them unfortunately. What 3 likes the most I think it is C.

And ask yourself whether C is owned by somebody. Okay? C is owned by nobody. So, he gets, claims C. Okay?

The next goes 4. 4 goes by.. Do you know what 4? E? Okay. So, E is owned by nobody. Okay? So, that's fine. Okay?

Now, for a 5. 5 likes A. If I remember correctly. Right? 5 likes A. Ask yourself if 5 is owned by somebody and then

so it's yes. In case the non-owner points to declares prefers the most. An object that is owned by somebody

actually moving up the serial order. So, instead of 3, 4, 5, so 5 is maxed. 5 points to A. A is owned by 1.

We moving up 1 here before him. We allow 1 to choose first. Okay?

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1 now chooses among the remaining object D I think. Right?

So, 1 points to D. Ask yourself if these owned by anybody? These not owned by anybody.

Then you allow him to make that pick.

So, the point here is that we run serial dictatorship, except that whenever an agent owns, claims, tries, attempts to pick up an object owned by somebody.

We move him up the ranking, the serial order of that owner. Okay?

What if you keep getting the same issue? Right? So, in other word is that let's say imagine hypothtical situation

well, let's say 1 and 2 are the owners and somehow let's say 3 points to A. A is owned by 1.

So, therefore moving up before 3. Okay? And that's the serial order next. Okay?

So, we let now 1 chooses make its choice. 1 suppose, chooses B. Okay?

B is owned by somebody. Right? We keep doing that. Then we will know about B. Okay?

What about now, let's say B now chooses A somehow. Okay? 2 owns B, but let's say 2 would like

B the most.

Then we have the cycle on the 2. So, until you keep moving up the ranking of the, moving of the serial order of those owners,

until you form of cycle. Once you form of cycle, then you resolve the cycle using like the top trading cycle kind of fromat. Okay?

So, these 2 mechanisms are equivalent, and this is what they call you request my house. I'll get your turn. Okay?

Essentially as he goes along this serial order, somebody claims your houses, now you get his turn. Okay?

He is moved down, you are moving up in the ranking. And then you also, that means you get to choose before he does. Okay?

If you also keep requiring somebodyelse's house, then he actually moves up your ranking. Okay? And then you form a cycle.

There has to be a cycle. Okay? Or else some in cycle want change. If you end up claiming house that is not owned by anybody, then you simply claim the house. Okay?

And so, this mechanism gives you 2 results I mean 3 properties maintain paretic efficiency strategy proofness and indivisual rationality for those agents for own houses. Okay?

So, these are the 3 different mechanism that we have looked at. Serial dictatorship, TTC, and sort of hybrid mechanisms.

They have strange name acutally URMHIGYT such like that standing for the using acronym, but this is a sort of ugly name. So, let's call that just hybrid mechanism.

The 3 different mechanisms achiving pareto efficiency and strategy proofness among other things.

Now, except for 1 issue, fairness. Okay? So, just think about serial dictatorship.

where, which position you have in the serial order matters a lot. Right?

So, the guy at the top of the serial order is the best off. The guy at the bottom is the worst off,

Unless everybody's preferences are all different. So, everybody can claim at the end the most prefer the object which is the possiblity you know preferences are all different.

Different people like different object. They end up getting what they like the most except for that situation the gut at the bottom of the serial order is really terribly worst off.

So, there is a significant fareness issue. And there is no way unless you can introduce monetary transfers in which case you can open market you can use VCG mechanism for instance.

VCG in my mind is a formal market. Right? You use monetary transfer as to fixed incentive problem there.

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Buying such introduction of monetary transfer which is difficult in many contacts. So, if you think about school choice, if you think about housing allocation for low income family.
If you think about office assignment in many different kinds of, or task assignment, many kinds of applications like this.
Introducing monetary transfers can be problem addict for whatever reasons like kidney exchange. You are not illegal to trade kidneys or human organs for instance.
The idea of selling public school seats goes against the idea of free education, free public education for instance,

that are constraints that you phrase that makes it impossible for you to use monetary transfers. So, what do you do?
To achieve measure of fareness, the best thing you can do is randomizing. Okay? So, exposed assignment will be unfair,
but, exante at least, you make the assignment exante fair at least. Okay?
So, the rotarys are used in many different contacts. And in fact, most of the school assignment that you know of, it is choice in assignment by rotary. Right?
You grew up a sort of very familiar with a rotary assignment.
The mechanism that I'm going to introduce what we call random assignment mechanism do better than simple rotary assignment..
So, remember what you went through in terms of really public school assignment for those you know went through this system.
They don't ask you what your preference side. Essentially you are randomly assigned to school in your neighborhood. Okay?
You can do better than that. You can ask individual students to produce submit preferences. And then you can try to do best.
At least in a random assignment. So, again, what you care about is some sort of efficiency goal and some sort of incentive goal and some sort of fareness goal. Okay?

So, we are going to call ordinal efficiency. So, ordinal efficiency is sort of exante version of pareto efficiency that

I mean so, exposed, we are going to when you have randomization, we will have to sort of think about exposed and exante efficiency issue.

Exposed for given realization of whatever randomization devised been using given realization of this random rotary,

you can then think about really resulting assignment is exposed to efficient or not in the sense of that there's no other assignment that pareto dominate to given assignment.

But, often when you use rotary, you also ask efficiency at exante level meaning that what you're doing at essentially is to is the you know assigning rotary.

You are giving each agent some probability of distribution of an different objects.

So, we have a collection of rotarys 1 to each agent over object. Now is that any way to realocate to property to share across agent, across object in a way that makes everybody happy. Okay?

That's the question did you asking. Here ordinary efficiency sort of like the exante pareti efficiency, but it's a little bit different.

I am going to have to introduce some additional notation and notion to talk about.

Roughly speaking an exante assignment is ordinally efficient. There is no other assignment that ordinally domanates.

The assignment and ordinal domination means that it improves the welfare of agent in the sense of first wood stochastic dominants.

In other words that for any object, I mean, we have to sort of build more notation here, But say that

Pi is an assignment allocation mechanism that [?24:23] from profile. So, this is a mechanism here. You know, let's not do that.

Let's think of this as an assignment here. That's the school rotary. Rotary is a probability distribution of.. So, you can think of this simplex.

as just called Lotteries

Lottery is a probability you can think of this simplex

probable addition of a different object

ok so this has to be one free agent right?

and that is feasible

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so let's say

f star is we are going to actually introduce this notion of fomally is ordinal efficient

if there is no other assignment pi

that such that B statistically dominates

first order statistically dominates pi star and by first order statistic domination

I mean that this improves the chance of getting chance of increase chance of getting more prefer object relative to pi star

so give me any object A ok?

and think about probability of getting an object which is weekly prefererd given relative duo preferences

weekly prefered to that object A

for any search A

pi'll gives a high assignment to get probability then pi star

I'll give you a more precise than definition

so all you need to think about at this point this is like excellent palliative deficiency

except this is like ordinal version

stretegic proofness is exactly the same except now

it's more involved in the sense that it's actually very strong condition

because we are talking in terms of the mechanisms that are assgined

mechanisms are the maps from the profile of ordinal preferences to the assignment here ok?

it's said to be stretegic proof

if the lottery that you get ok?

for each agent

ordinally dominates the lottery that you will get

and lottery that you get when you tell the truth

all ordinally dominates the lottery that you will get when you lie

ordinally dominating again meaning that lottery you will get when you tell the truth

first of the statistically dominates

the lottery that you will get when you lie

the statistic domination I will define more carefully

basically improve the chance of getting more prefered object that's sort of the sense you should have at this point

these two are kind of the well fair and incentive property you may want to achieve

this is the fairness very minimal fairness requirement

it's often called equal treatment equals

meaning that those who submit the same of ordinal rankings

must get the same lotteries ok?

if you think about any serial dictatorship ok?

you can easily see the c netries violated

I'm not talking about random serial dictatorship and talking about determinite serial dictatorship

I haven't introduced random serial dictatorship yet

so think about a serial dictatorship with any given serial order

let's say the first guy and the last guy have the same ordinal preferences ok?

now the lottery is new end of signing the two of them are not the same

lotteries are all degenerated

degenerated in the sense that's ah it's not lottery at all

so the first guy will get the most preferred object

the last guy will get whatever is last for him

so lotteries are not the same

so can kind of see why this is important

when this is a kind of reason while you want to introduce random of assignment

these are the 3 goals

one representing welfare, one representing incentive and one representing fairness

you may want to achive 2 or together unfortunately

it is impossible to achieve all 3 of them together

that's the fair very famouse result

for any agent and great number of great agent are equal to four

there is no mechanism in their achieves or 3 goals

and there is also cardinal impossibility result

which these are the 3 different with cardinal mechnisms so

we are focusing so far on ordinal mechnism where the preferences for agents are ordinal

or we only care about the ordinal rankings that may be actually cardinal preferences

we just think about the ordinal implications for ordinal rankings

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when we think about cardinal mechanism you can imagine that

mechanism where each agent has Von Neumann-Morgenstern values for each object

and the mechanism may ask each agent to submit that Von Neumann-Morgenstern utility values.

ok? And try to assign lotteries as a function of those Von Neumann-Morgenstern values.

so much more richer preferences are domain

so the mechanism, cardinal mechanisms actually work with cardinal messages like that

and the welfare notion should be also different right?

welfare notion should be standard excellent palliative deficiency

stretegic proofness should be also different in terms now it'll say not ordinal domination

you know simply that reporting truthfully about your Von Neumann-Morgenstern utility values

it's a redominat stretegy ok?

semetric view also dislike different right? people who two agents report the same Von Neumann-Morgenstern utility values.

we'll get the same lottery

so considerably think about the cardinal versions of these 3 properties

and there is no mechanism that are achieved or 3 of them ok? So

these two results are impossibility results of available and in terms of that

one doesn't imply the other there are not really related

what a think that the cardinal implies ordinal it turns out that's not the case becase here ordinal stretegic proofness is very very strong in fact

so this is natural extension of serial dictatorship

go random serial dictatorship

so only do here randomize over 3R orders

ok so how many possible serial orders can there be if there are an agent

suppose that an agent how many so

this is the cardinal

small n is cardianl

how many serial orders can there be

n factorial so

each serial order is chosen with probability

one of n factorial ok?

and then given serial order we want serial dictatorship

so x suppose unfair because there is their top guy and their worse guy

day and night difference between the two of them in terms of what they will get

however you know everybody has a chance of being a top guy

so in that sense this is a fair

the nice thing here is that we get x pose deficiency of course

which is exact what we prove

for giving realized for giving serial orders that is realized

the assignment you get is palliative deficiency

exposed it's also stretegic proof ok?

it's also semetric in the sense that you end up getting the same lottery ok?

because it is realized in the same way so if you have the same to take two agents whose ordinal preferences are exactly same ok?

they end up getting the same lotteries

serial orders completely reform and randomly assign determines ok?

another way another quite interesting result is that

another mechanism we can imagine

which set of we can think of extension of ttc is the following

so you first of all

n object n agent ok?

then you can randomly assign the objects to agents

subject to the constraint that each agent gets one object ok?

and think of them as the indolent and then learn ttc

you then get exposed efficiency and also stretegic proofness

but then now we are randomizing over ownership right?

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uniform randomly assigning ownership

and so you will guarantee also sematry as well

another way to regenerate semetry stretegic proofness and exposed inefficiency

as it turns out this is the paper

this shows that lottery that assigned as a function of the preferences ok?

are exactly same between the two

these are two different mechanisms

and yet as a function of the profile of preferences

the lottery that you determine for each agent is exactly same between the two ok?

so two different way of randomizing and they all lead to the same randomal assignment at the end of the day

that's kind of interesting however and this will be the last bit for today

is that the two mechanisms let me just focus on random serial dictatorship

actually random serial dictatorship is quite widely used

you naver see it as a participant or we are doing is you ask to submit ordinal ranking

you rank the objects you rank different schools in school choice

you rank different extra curricular activities when you play this extra curricular activity assignment

a mechanism which is common in after school assginment and so forth

you never see what mechanism is actually used

for most of the cases what's uses the random serial dictatorship

all you do is submit your ranking

so you do not no exactly what mechanism is used in fact what's often uses random serial dictatorship ok?

so they actually randomly generate serial order and then do this serial dictatorship

and make sense because it's sort of has nice qualities

so you can say that the random serial dictatorship is the most commonly used

random assignment mechanism

turns out it is not efficient

exposed deficient that is not efficient

here is the example and this will be the last slide for today

let's say that there are two objects A and B

here the number of objects do not match up the number of agents

but that's not a probable

we could have added two additional objects C and D which are very very bad

so and then suppose that there are 4 agents 1,2,3, and 4

1 and 2 like more than B and B more than having nothing

3 and 4 like B more than A and A more than having nothing ok?

now random serial dictatorship or each called alternatively known as random priority roll

so when you say random priority roll that just means random serial dictatorship

here are the lotteries

1 and 2 each gets lottery according to the each gets A with probability 5 or 12

B with probability 1, 12 and nothing with probability one half

this is the matching mechanism as we said

and there are two objects and there are four agents

so it's not supprising that each agent gets probability one half

each agent gets some with probability one half

but he gets less prefererd two with positive probability ok?

same goes here so they like the most prefererd one with fairly high probability

but still with positive probability

they also get the less prefererd two

now you can imagine that they can swap these two

each of that swap this their probability share of the less prefererd object with the other one

so get rid of this in exchange for additional share of A

and as a result of trading this probability of shares

they may arrive at this lotteries and this is better ok?

now first of all let me explain Tu why this is the case

why each gets the relaxed prefererd of the two positive probability ok?

now you can imagine the following serial order 1,2 and 3,4

so there are four factorial different ways of determining coming up with serial order this could be one of them

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we've chosen with this probability righh?

so one sixteenth

now in that case, one has the top position

he choose A because he likes A the most

A and one is gone now it's two turns

and two has only B remaining and he likes B more than having nothing

so he ends up choosing B ok?

the incidence in each B chooses 2 chooses B of course with positive probability

it's not surprising that this is positive probability ok?

so everybody this part of the dominates that

even though we haven't assgined Von Neumann-Morgenstern values.

as long as you assign Von Neumann-Morgenstern values that are consistent with this ordinal values

in other words that you V of A for agent 1 of having B

assign any values prserving this ordinal rankings

this is better than that

regardless of what cardinal values agents have

as long as they are consistent with this ordinal ranking

this part of dominates that

and that's actually what written by ordinal domination

the most specifically I'll say

so fix any other object A ok? The probability of getting object A more better is in this case one half

in this case something must one have ok?

so and then let's fix up object B the probability of getting object B or better

is one half here still one half ok?

so in that sense the probability of getting your most prefererd object is higher here than here

in the probability of getting the second most preferered object or better is higher

or greater than or equal to the probability here

in this case that's the other same

so in that sense that's the definition notion of this lottery first of the statistically dominating lottery

I'm going to introduce the formal definition but that's basically requirement

and that requirement is satisfied

so in other words that lottery here

that this mechanism this assignment first of the statistically dominates that assignment for each agent

so in that sense when that happens we say this lotteries ordinary dominate that lotteries ok?

so this is the illustration of ordinal inefficiency of random serial dictatorship

that assignment you get from random serial dictatorship maybe ordinal inefficient ok?

so that's the end of today's class

I'm going to talk about ordinal efficiency next time

I'm going to talk about mechanism that is designed to fix this problem ok?

something called probable realistic serial mechanism

so I'm going to distribute these slides for these lectures and also this additional servey paper by [?44:8]

that I promise I'll distribute as well ok?