Title: 시장설계이론1, 이산 자원의 분배 (1)

- ✓ **Instructor: 최연구**
- ✓ **Institution: 연세대학교**
- ✓ **Dictated: 강은경, 강성호, 김신희, 김종백, 신원대, 현소형**

[00:00]

with the provide you with lecture notes, but apparemtly I given.

so I gonna, after this class, I'm going to..

so make this, make this slides available to you.

hopefully

not having the chance to look at this beforehand.

should not be a problem.

I don't tnink it will be.

because it'srelatively sort of easier more accessible.

I think material that I'm goning to do today.

so , so far just give you a basic overview.

I mean, I'm just give you an idea about where we are.

so we started out, sort of giving you some road map about the matching theorem.

and what we did so far is just two sided matching.

so problem where that agents on both sides and the issue is how to match agents on one side to agent to the other side.

we started out a very simple structure with a one to one matching, one agent on each side gettinf matched with a one agent on the other side.

and then we extended the framework to many to unmatching.

we would simple preferences called responsive preferences.

and then we subsequently relax that the assumption to allow for general form of substitutable preferences.

and then finally, sort of the state of the art that we discussed last time was the Hatfiel milgram model which does the same sort of generalization in terms of preferences but also allows for contracts, okey?

ant then we show that the main result that are two of them I mean, few of them.

but you know the existence of stable matching and some of the properties of the stable matching as well as the incentive properties, actually carry over to this general framework.

now to what extend we can push the envelope to generalize has been sort of studied quit extensively by the literature.

Hatfiel milgram is certainly very important paper in that regard.

but there are other further attempts to generalize which I'm going to talk much.

but just to going to you know mention.

so Hatfield was involved in a number of projects.

but before that, let me Ostrowski so for those who are familiar with a sponsor search auctions, he is one of the authors.

this guy is simple just B.

he has a paper you know, in a year which is sort of tries to generalize the you know, framework to include sort of, I mean, address framework beyond two sided matching.

so think about situation where that are number of traders in the market.

and each, let's say from hire somebody who will provide input.

but then that supplyer also in turn hire somebody who will supply even more premative input into him.

so we can sort of imagine more general network structure in that.

right, so this has multilateral structure.

in the sense that you know, that is one side you know, which manufacture some products to be supply to consumers and

that is intermidiate goods and then that is further inputs and so on.

now this is much fairly general network structure.

and we sort of don't expect the nice result.

so we have to generalize to more general, I mean, like general structure framework like this without any restriction.

of course we, I mean even with two sided matching we needed to impose some restriction.

we need to put some coditions which is you know, stability, substitutability.

so you need to have some sense of substitutablitliy still to hold.

in addition, a network structure need to be restricted as well for the framework to work.

work in a sense that the stability can be maintained and so on

so the relationship that they needed to assume was some sort of acyclicity okey?

so framework must involved relationship could be like this, that like this.

but, but then doesn't have any sort of direct relationship, you know giving back to them, okey?

so that's one frame again that frmework has been also studied you know work by Hatfiel terminals.

[05:00]

so that's already published In a year.

this is not published yet.

but try to sort of show the neccessity of the framework to guarantee existence of stable matching.

so I think that this is kind of more important work that you know it's more useful to know that on the some framework, you know, things work.

here it's a little bit more of interlecture exercises to show that that's the most we can push okey? some sense.

and then there are having some work by, work by Hatfield and [05:40]

I'm talking a different paper here.

trying to, trying to expand a framework to include many to many matching, okey?

and there is another paper by [06:03]

and I think [06:08]

so those are the papers that assume that try to show existence of stablitiy.

so we don't expect the one sided strategy proofness to result carry over.

because remember that even in with Hatfiel Milgram one sided strategy proofness could only work for the side that match

with the one party, one agent on the other side like on the doctor side.

you know, we..

so that agenda is a little bit dropped here.

but the question is really one kind, I mean, can guarantee existence of stable matching.

now if you, if you remember what we did, what us really key assumption assumption in guarantee existence of stable matching under Hatfiel milgram.

you remember that I write was importance was substitutability.

and what was the role substitutablity of preferences play, the role it play was to gurantee monotonicity of the operator.

if you remember this R function rejection mapping.

and so for the hospital side, this rejection mapping, right, was monotone which was important.

fot doctor side, the fact that doctor can match with only one hospital.

each doctor can match with only one hospital.

imply that, you know, substitutability is kind of automatically satisfied.

now it seem,at this point, is not clear that why needed to assume that instead of assuming more generally that doctor's preferences also substitutable.

doctors can also match with many agents, many hospitals.

but their preferences also substitutable, okey?

so this basic framework the existence of a fix point follows from tarski's fixed point theorem, which requires nothing more than just the monotonicity of mapping.

so therefore the framework, you know, carries over very smoothly.

I mean when you drop this feature that doctor can only match with one hospital as long as you maintain substitutablity on the a side.

but the notion of core and the notion of stable matching becomes a little bit more problematic.

and that's weird.

I think that the research here is, has been focus on the..

that simply guarantee fix point, it was not our problem.

and also showing that the fix point corresposive to stable matching is not also problem.

problem is the notion of stability which is a little bit tricker with many to many matching.

so that's kind of the state of art, okey?

now, so in terms of application, two sided matching has been applied to medical matching as we know.

but to a degree, has been, once we say, has been applied also to school choice which will discuss.

but school choice has, a sort of, a number of aspects, okey?

that's kind of requires a different sort of perspective that is you know, when you talk about school choice.

eventhough I mean, both sides sometimes have preferences.

students have preferences of schools.

and schools do habe some sort of priorities about which student they like to admit, okey?

but especially when you are talking about assigning students to public schools at the level before college, okey?

it's it's not clear that we should take the college, the school side as a strategic agencel

[10:00]

I mean we talk about college admissions really.

and then clearly colleges are, you know, sort of autonomous.

and they have their own preferences.

and they play strategically.

and so therefore two sides matching is kind of the right way to think about it.

but if you think about assigning students to schools at the level of below college.

so high school, middle school, elementary schools

it's you know, in some sense that this is more like a problem of how to sort of maximize the welfare of students

taking students as resources essentially I mean.

so you should sort of treat this seats are available from different schools, some sort of resources, okey?

so in that sense I think that framework that I'm going to introduce today will be more important.

namely that this sort of one sided assignment problem where you sort of care about the welfare and incentive.

so only one side, okey?

but at the end, I mean that a some sense in which that well also apply two sided matching when we talk about school choice as well.

because even at that level well at the level of high school, middle school that are some public policy reasons to worry about school's priporities about student.

that are some social welfare goals that are sort of requires different schools to prioritize students in a certain way, okey?

so it's gonna be a mixture of one sided assignment and two sided matching that will apply when we talk about school choice.

so I'm hoping to do it, start talking about it.

may be starting wendsday even, but certainly on Friday which is the last class.

so here the basic framework is very simple that are finite number that is finite number of the finite number of agence.

so N is kept, N is that rotation used to denote the set of agence, right?

and each agence demand at most one unit of goods.

and that is a fix that is a fix set of goods which is also finite that are finite sort of items that are available to be assigned to studnets.

and that are in general that is also possibility of assigning no good to an agent, okey?

some agent may be unassigned.

so that's is denote.

so the sort of will introduce something called null good to accommodate the possibility of unassignment.

and o tilde

it is still used to denote the union of [13:04] null good and the proper good, okey?

and each agent has trick a preference pie I.

pie I you can think as a ordered list of goods just as we did over o tilde.

so

okey.

so first of all we are going to sort of start with what we call [13:35] assignment.

later on we are going to take an issue with what we call deterministic assignment.

because deterministic assignment means that you know when assigning this objects to agents.

that is no randomness involve.

we are not using lottery for any reason, for any face of the assignment.

and that is a reason why that may be that may not be desirable, okey?

but I think this is the sort of the..

we are gonna start, I mean, we are gonna build a framework sort of gradually.

so this is the easiest case, simpliest case.

so there are ways, we've think about deterministic assignment meet that simply means that well,

you assign this objects to agents one for each agent and at most, okey?

so that's every such assignment is feasible, okey?

you are not gonna assign more than one good to some agents.

subjects to constrain anything else is actually anything else is fine.

and so we usually care about two goals ,right?.

one is pareto efficiency.

and pareto effociency here means expose pareto efficency.

it is in general means very clear in the..

once you focused on the set of deterministic assignment as long as that are no randomness involved.

[15:00]

that is no distinction between ex-ante and ex-post in fact.

pareto efficiency means the following I mean usual notion.

so take any given feasible assignment.

that assignment is pareto efficient.

if it's not pareto dominated by another assignment.

then pareto domination is defining the usual way.

namely that one assignment dominates pareto dominate another assignment.

if the former makes everybody weakly better off relative to the letter and some strictly better off, okey?

pareto efficiency is usually one goal.

strategy proofness is another useful goal.

strategy proofness here means that the way.

so again we define assignment and machanism separately.

so assignmnet is exactly the way I described, right.

so, so if you give me an identities of the agence, okey?

some index identify the different agence.

than as a function of the identities we sort of, you know, describe which good must be assign to which agent.

okey? That's assignmnet.

machanism here is a little bit more complicated object.

it's a mapping from profile of preferences, okey, into assignment, set of assignment.

if give me profile of preferences, one for each agent.

then I'll tell you exactly which agent must get which ob, okey?

and then strategy proofness simply means that the machanism just describe just before.

it is strategy proof if it is dominance strategy weakly dominance strategy for everybody to report truthfully

about the ordinal ranking in this case.

depending upon the circumstances, of course that the preferences can be ordinary preferences can be cardinal.

I mean we don't know exactly what I mean by cardinal yet.

but it will be clear when I talk about it later.

so so far preferences are ordinal.

so what each agent need to report is the ordinal list about objects.

and as a fuction of that profile of ordinal list of objects.

then a machanism determines which agent gets which good.

and this way of assigning in the sense.

this machanism is strategy proof.

it there is no agent that has.

if every for every agent it is a weak dominance strategy to report truthfully given the machanism.

and there is a machanism that achieves that, okey?

it is serial dictatorship, okey?

once you sort of think about it.

some resemblence here of the[18:00] result.

in the sense that I mean desirable set of machanism see sort of reduce to dictatorship.

here serial dictatorship a little bit different.

in the sense that it's not actually necessarily I mean, we think about dictatorship.

dectator's preference is maximize at the expese of everybody else.

serial dictatorship is not as bad.

okey, so what is serial dictatorship?

we assign serial order to everybody, okey?

so you are number one, you are number two, you are number three and so on and so forth.

you kind of line up agents in single column in some sense.

and then the first agent goes and picks the best object to him or her among all objects, okey?

and then he exist with a good, okey?

and the second guy the line actually makes the pick okey?

choose the best one among the available ones and so on and so forth.

that's kind of the very simply what the serial dictatorship how the serial dictatorship works, okey?

of course, the way we will implement this machanism, okey?

we don't need to in fact have agence lined up physically, right?

so we just collect the rank orders, right?

preference rankings of objects.

and then we can simulate the outcome that what will happen that what will occurred, right?

the assignment erruted in implemented had the agence will lined up according to serial order and so on.

I mean I make them pick right?

simply if you given the serial order, serial order need to be decided first, given to implement this machanism.

If you given the serial order, serial order need to be decided first given to implement this mechanism.

Given the serial order, we just pretend that the guy at the first top position makes the best pick by simply asking little viral his preference ordering and review picks the best object according to the preference ordering for him.

[20:00]

And then we sort of do the same thing recursively.

There is a question that how to come up with the serial order.

But, given one of serial order, we can show that the resulting outcome is pareto efficient.

How come? So let's say that phi is a … So this is a mapping from agents to objects.

It's an assignment, feasible assignment.

Any mapping here will give you a feasible assignment essentially.

This is an assignment, not a mechanism.

And suppose that an assigment associated with some serial dictatorship given some serial order.

In fact, I mean, we could have made our life easier by saying N is described by numbers, natural number 1

through right of the cardinal

And think of each number as the position of that agent.

So we can do it without any loss.

We simply winned that X agent by the serial number.

One thing that we can show is the following.

Then, in that case, if agent here … this is some object … he get some object for each I.

That's the assignment.

Now suppose that … so, to show that the phi star is pareto efficient, that's the claim.

By the way, mix the term pareto efficiency and pareto optimal at the end, and they are the same, of course.

I am going to show the following.

I am going to first define … suppose there is sum assignment, alternative assignment.

Suppose this phi weakly pareto dominate … so emphasize weakly here.

Weakly means that we are not … this is different from the standard notion of pareto domination.

Standard notion of pareto domination requires dominating, this to dominate weakly for everybody and strictly for some agents.

We are just dropping this requirement of strictly.

For instance, phi star itself can weakly pareto dominate itself.

So if that is the case, what we want to show is exactly this.

If we show this we are done.

Because anything that weakly pareto dominate phi star must be itself.

That means that there is no phi that pareto dominates phi star.

[25:00]

When we say pareto domination, we also include strictness.

So it's very easy.

What's the argument here?

So first of all, argument is that, we show that this must be the case, right? Why?

Because the guy, first guy gets his best object.

So anything for any assignment, alternative assignment to be weakly pareto dominating phi star,

the first one must be getting exactly same thing, the frist, I mean the best.

Otherwise, it will be strictly worse off.

This is also quite clear.

Assuming that phi assigns same objects to agent one through came in S1 agent.

Therefore we lose degree of freedom in terms of assigning objects for those guys.

So you are basically removing all the agents one through came in S1 along with what they get on the phi star.

Then the next guy, right, unlying must be getting the same object as he would on the serial dictatorship.

Why? Because if you get rid of all this assignments to agents one through came in S1, the objects that is best from the perspective of the next stage K is what is getting on the serial dictatorship.

So if you imagine any pareto dominating reassignment, weakly pareto dominating reassignmnet, it must give0

the same object to the first guy.

Because it was the most preferred one.

The next guy must be getting the one that is best from the remaining ones.

So that's how we show this.

Strategy proofness is also not very difficult to show.

Because fixing the preference profiles of everybody except for any given agent

So in particular, those who make picks before him, their preferences are already chosen.

So that means if you run serial dictatorship.

No matter what that preferences are for those guys, once you fix the preference of all those agents except for any particular agent I.

The assignments to them are determined by the profile of preferences that they submitted whether they are truthful or not.

So basically, lying cannot pay for this agent I, because he will be getting the best object among the remaining objects.

Remaining after getting rid of all those objects that are taken while the agent move before him.

So therefore honesty is the best policy in this case.

And that is true regardless of the preferences of the submitted by all the other agents.

Once you fix those, you'd dare as well by telling the truth

[30:00]

So we have shown that this simply easy way to get pareto efficiency and strategy proofness

Furthermore, you might ask the opposite question, the converse relationship.

Now suppose you want to implement a particular pareto efficient assignment.

Thus there exist a serial order such that if you run serial dictatorship using that serial order, we can implement

that given pareto efficient assignment.

This is sort of homework question for you, so you should think about it.

Give me any pareto efficient assignment, I can find you a serial order such that if you run serial dictatorship,

you will get that assignment implemented.

The other thing that going back to ?[31:06]

If you imagine that there are two objects, A and B.

And I like A, and she likes B.

No matter who moves, we get the sort of the first best outcome.

So serial dictatorship I told you is not as bad as just dictatorship in some sense.

There are many different outcomes, different assignments that can get implemented depending upon how you choose serial order.

This is similar to having different kind of assigning, basically designating different agents as dictators in the ?[32:08]

The reason that we think of [?32:13] dictatorship as really bad, not very fair to the agent is because …

I mean here, there is some issue of fairness as well if you give a fixed serial order, of course the first guy is the best of,

and the second guy is next best and so on and the last guy is the kind of worst of.

That is true, so there is fantasy issue which motivate us to think about randomizing of serial order later on.

But still there is a room for an outcome that is not so bad even with a serial dictatorship.

Because we are implicitly imposing some restriction on preferences

So remember here that the agents care about what he gets.

They don't care about what the other guys get.

So there is a lot of degrees of indifferences.

So if there are three objects and as long as I get A, I don't care whether second guy gets B or C.

You can imagine two different assignmnets where the same guy gets A.

There are three agents, let's say, one, two, three, and there are two different assignments where agent one gets A.

One assignment is where agent two gets B and the next assignment is where agent two gets C

Even assuming, making assumption that in every assignment, every agent gets some object instead of no object.

Between these two assignments, agent one is indifferent.

So in some sense, we are exploiting a lot of indifferences to come up with an outcome that's not so bad in some sense

from the perspective of fairness.

The first problem that we handled is known as house allocation problem.

House allocation problem … I think that the origin is the Hillan Jackhauser paper which we are going to talk about a little bit.

So house allocation problem is simply allocating and houses to agents.

In an environment when no agent has existing endowment, doesn't own any house in the beginning.

That's house allocation problem. That's what we did.

Second problem is known as housing market problem.

[35:00]

I think the origin is Hillan Jackhauser, here the origin is Shapli Scope.

I think these papers are from the 70s.

Again here that I say number of houses as the number of agents that N house is N agent.

But the problem here is slightly different in that the each agent has initial ownership of house.

Each agent is endowed initially with one house each.

And the question is really can it reallocate the ownership in a way that makes everybody happy.

So the goal here is therefore similar.

Can we achieve pareto efficiency at the end of this reallocation of houses?

In particular in a way that is strategy proof.

And the answer is yes.

And just as in deffered acceptance algorithm, Gale &Sharply, Sharply's curve also propose constructive algorithm that can be used to produce a strategy proof and pareto efficient reallocation.

It's very intuitive and everybody likes this mechanism, it's called top trading mechanism.

What you do here is actually the following.

Again, if you run using like computer, all you need is to have initial input, information about who owns what house.

Plus, the ordinal list of preference rankings of houses.

But it's more interesting and easier to understand if you run it just like a serial dictatorship.

Here is how we did.

Each agent points to more preferred house.

So if you run it through in the proxy version using computer, we are basically computer will point on behalf of a given agent to a house that's most preferred according to the preference list submitted by that agent

And then each house points to an agent that owns that house.

Agents point to house, and house points to agent that owns that house.

And then, we form a cycle based on that.

Actually it is kind of the equivalent, even easier to think of agent not pointing to house, but rather agent pointing to the owner of most preferred house.

In that case, agent will point to another agent.

So basically start from any agent in A which points to another agent who owns the house that's most preferred to agent I

Agent I points to another agent who owns the house that's most preferred to agent A and so on and so forth.

Now, each agent points to only one agent because you are assuming strict preference, strict ranking preference.

And then there are finite number of agent

So at some point there must be at least one cycle.

That could be multiple cycles.

any agent can belong to at most one cycle. Ok?

and there must exist at least one cycle which comes from the finiteness of number of agent, ok?

now, suppose that this was the cycle , then what we will do is , here is what we will do

[40:00]

ah, two agent i we assign the house on by J

so each agent , ah , in their cycle is assigned the most preferable house

assigned house most preferable to their agent that will step

this is sort of extension of the vital economy right?

so, suppose that I points to J and J points to i

this is a very simple form of cycle , actually there is even easier simple form

of cycle which is exactly this right? I points to himself which is possible ok?

meaning that he owns the house that is most preferable to him

so then , in that case, implementing cycle means that you assign that house

to him permanently ok?

here , that means that they swap houses ok?

so, you can but imagine a little general form of cycle of this kind ok?

so we call , we say execute , executing cycle ,ah do mean exactly what I say ok?

this extended , um, swaping along the cycle ok?

by giving the house is almost preferable to the agents on the cycle

and then what we do , we remove the agent along with houses that are assigned to them ok?

so then, we start with a big problem consisting of the set of agent and set of houses ok?

we get rid of some agents and some houses , same number of agents and number of houses essentially right?

but remaning problem is just before similar to what we have before , that we had in the first place

except that ah, the number of houses and number of agents are reduced by the number of agents and houses that are removed equal to the number of houses and agents that are removed in the first step

now and then we can do the same thing over and over again

since in each step , there must be at least one cycle ok? So therefore the number of agents and houses get shrink,

shrunk in each step strictly ok?

and therefore this algorithm terminates in finite steps as well

(student speaking)

sure yeah, so this is an example so if that is only cycle, ah, in fact that it cannotbe , that could be only the cycle

because if everybody points to I including himself , that's the only cycle that will clear is this one ok?

then the, in the next step, we have one fewer house and one fewer agent

and at least one person and one house will drop in each step

so at most and step is necessary to run this algorithm

usually, you will do better than that , you will do finish , you will do finish up faster than that that

because the cycle normally will be more , will involve more agents than just one

(student speaking)

sure yeah, so what does the pointing reflect , pointing simply reflects the preference ordering ok?

ordinal list of houses that the agent admit

so if you use, want it using this computer of course the computer will do the

pointing for you , for the agent essentially right?

basically look at the list of preferences and see really um,what was the most

preferable object to each agent

and we will just form a graph ok?

and the next exquisite cycle and you know, recursively it right?

so um, ok, now

so this game, an easy problem , just simple example

so one in this case if you do sort of second version of pointing one points to whom , one points to , the assumption is that 1 owns A ,2 owns B 3 owns C ok?

so 1 points 2 right?

because 2 owns B which is most preferable one

[45:00]

and then 2 points ,what, A who owns A into 1

and then 3 points C and 3 points 2

because B is owned by 2 ok?

it is possible that you may point the , more than one agent of course ok?

but you point only one guy

so, in the first round this cycle is formed and exquisited meaning that 1 and 2

struct the houses and they drop out by the end of ground

the second step, ah, 3 , now the problem is reduced ok?

so the problem that doesn't include house A and house B ok?

so there is only remaining house is C ok?

so therefore, what we need to do, so you drop , eliminate B and A ok?

and the only question that you ask is whether C is acceptable to 3

and in this assumption, every houses are also acceptable

in the original shapley's curve problem old houses are acceptable

you, whether own some house , then not owning any house

but you can easily accommodate, ah the possiblility some houses are not

acceptable to you

in each case, you will simply point to nothing because you allow null good to

be part of the good in this case

so therefore you will end up not assigned in that case

um, the strategy proofness hold in fact , we reduce this mechanism , mechanism here is strategy proof . It's dominance strategy for each agent to report truthfully this is probably they are most involved , proof but it is not actually difficult to

understand why this is strategy proof

so one important observation here is that , it is paper by   loss in economic letters

which first prove this result but then there is sort of easier version atdikadrabler someas in 2004 , 3 might be actually , 2004

it is school choice paper and they actually reprove the result, they either show me extension of , thet extend top trading cycle's mechanism in an environment where in a multi unit sort of many to one mating sort of enviroment

we will talk about it later so you don't have to worry about

but the idea is that proofness is simple so suppose that your preferences Pi

you are agent i

and you are, this is your true preference ok.

and suppose that you fix so this is order list of the object for houses, fix everything else

so Q not I is just the profile of preference list submitted by every agent except for agent I . We just fix them arbitrarily

so, right?

and agent I contemplate now deviating truthful reporting by lying about it

and suppose that he does this ok?

suppose that by reporting truthfully he gets object , some object O , O *(o star) whereas by lying he gets some object O at step T * and here at step T

[50:00]

and we are gonna show that this some of lying cannot pay strictly for this agent ok?

so first thing to observe is that um. At the beginning of step ok?

same set of agents and objects remain regradless of whether reporting Pi or Qi ,

regradless of whether reporting Pi or Qi   the same set of object   and agent remain at the beginning of minimum of this two ok?

of course, let's say that T is smaller that T* ok? That means that at step T

here get assign object O , so therefore from then on the allocation of difference

but before then, the allocation actually that gets assign , the house that are assigned and the agents that are assigned will remain exactly same

why is that? Well because that because the set of cycles that are formed ok?

prior to this step cannot change ok? Between the ,so no matter what you submit this or that ok? The set of cycles that are formed and exquisited cannot be affected by that

it is quite easy to see in the first step because so you are agent I ok? So agent I here and there maybe some agent , maybe pointing to you

but the fact that in the first step , let's say this is not the first step ok?

in the first step , you are not part of cycle ok?

what that means is that the every other cycle here that is formed let's say somebody else form a cycle like this ok?

um, that will involve you , and we are fixing the preferences ok? In both cases ok?

and so any cycles that are formed in the first cycle cannot be affected by you decision between Pi and Qi in the first step

and they will be formed , they will be removed ok?

given that same thing can be set in the second step and third and so on ok?

untill you find yourself in the beginning of that step ok?

and then so therefore in both cases, you are dealing with same set of agent

and the same set of object at the beginning of that round

there are two possible cases of course

one case is , this is possible

now, so you find yourself you are at step T ok?

now, you report , so if you do the pointing according to Qi , you get O of course ok?

[55:00]

so at step P, if you did pointing according to Qi preference Qi you will be about to know

but if you suppose you didn’t , let's say that , so let's say O is owned by some agent K ok?

so we point K at step T and you will get this object ok?

so you can think of all the agents and the object that they own , they think of them as your opportunity set essentially right?

they are always pointing 2 as well as you are remaining ok?

so therefore , they would not change their pointing ok? Unless you drop out ok?

ah and you point K and then form a cycle ok? And then you get O ok? That's one possibility

suppose you don't do that ok? You point Pi accroding to Pi, wjhat happens?

this guy remains, they will never go away . They all keep pointing in such a way that the moment that you point K ok?

ah, you can form this cycle ok. So this opportunity is remain available , they don't disappear as the step's progress ok?

so what you do is that therefore as time goes , your opportunity set grows

all people make point to somebody , to you and different cycle that may be

formed

but existing opportunity is , I mean sort of defined opportunities mean the older sequence of older pathes going through pointing to you in the end, they all remain ok?

and in fact, not only remain this opportunity set of pathes grow over time

and at point T * , you get essentially the best object in the set of remaining object ok?

so therefore you cannot possibility do or O by pointing to accroding to Qi and

claim O at this point because at   time T* ok?

you will get the best object that is available . The available set up object include anything that isn't available to at T ok?

so that's the observation that is key

finally, suppose this is ture ok

well, here the way to think about it here , let step T*

you are getting the best object that is available at this point

you cannot do possibly worse by postponing ok?

so it is possible oppose that you postpone and claim the same object later ok?

that you cannot strictly better

ok that's the basically the idea behind the claim , this one pareto efficiency of

trading cycle if in fact , easier to understand this is actually homework problem , let me just go and argue

it's very similar to the serial dictatorship

the same claim that we prove for serial dictatiorship right?

so suppose that Pttc is the mapping you get

So, suppose that, Pttc is the mapping that you get, it is an assignment you get from TTC, Top Trading Cycle.

And, let's say that, let's partition the agents into N1, N2, through Nk.

When Ni is the set of agent assigned at step i

[60:00]

So the number of agents that are assigned, in the first step of TTC, the number of agents that are assigned in the second step of TTC, and so on.

So we know that the algorithm ternimates in the final step, and k is the last step.

This is a partiton right?

So there is no interception of agents in the different set.

And the argu is simply, suppose that, B weakly pareto dominates,

Then you can see that for any agent, I should say I, so maybe, let's go to the j, here, any agent I,

This is the first step of argument, meaning that any agents who are assigned in the first step, in TCC

What are supposed to them?

They get in the most preferred object.

And so therefore, in any pareto dominating, at weakly pareto dominating, we assignment, if you imagine, they cannot get anything else.

or else they will be strict or self

So they must begin, exact same thing, okay

So then, that's the first part of the argument, second part is induction clearly,

The same is true , for all I, N1 through N, should not use k there, k-1,

Okay this is different K there now,

Must be true for agent I in the set Nk

Because, if every agent in this set, is assigned exactly same object, as he or she would be on the TTC,

That means that we have already lost of degree freedom in terms of assigning what object return sign to any agent are being remaining.

The particural agent belongs the set, we cannot assign object that are going to sign to this paper.

Remove them, these obejects,
And Remove the agents that are assigned, looking at basically assigned the best object that are available at that point to pick here.

And here, I mean the TTC, each agent belongs who belongs the set is getting assign, the best object.

That is availablele, after taking out all the objects that are having assign.

Therefore, any in order to pareto improve at least weakly over these guys most beginning excatly same thing.

And the argument inductively can be used this way to show the assignmemt this pareto efficient.

The some interesting characterization, so I think, let me just emphasize first for this one.

Here, term characterizing, what characterizing means [?1:4:28 actual]characterization

So, often, this is sort of very fashionable in the context of matching series, in the context of the co-operate game theory in the context of decision theory is that,

Suppose you have some sort of mechanism, TTC, okay,

Now we know TTC has certain property, pareto efficiency and strategic proofness.

But then, you should have ask question in the opposite direction.

Suppose, you are looking for your searching from mechanism, which could be TTC, which could be something else.

[65:00]

Satisfy this properties, pareto efficiency, and staregic proofness, other anything else.

Clearly, you can imagine there is a different way to go assigning but pareto efficient, strategic proof, we already did all the one, in addition to TTC, like we run serial dictatorship, with arbitrary serial order

Ignore the ownership structure, don't repect who owns what.

Just line that any arbitrary order, one through one the serial dictatorship,

The assignment you end up getting in, it will give you some assignment, the assignment will be beginning is pareto efficient and strategic proof

So we already know that there are mutiple ways of assigning, a multiple mechanism.

That satisfy these two properties.

So if you ask this question, the opposite question, are you end up getting multiple mechanism,

So when you say characterizing particular mechanism, you add sufficient number of properties, so that

The only mechanism satisfying this list of properties you insisted upon is unique.

So when you say TTC, characterizing TTC, you will be basically proposing the number of actions

Such that the only mechanism that satisfy number of action is TTC.

It turns out that you can accomplish that goal, by adding just one more property.

And that's quite , a property that is quite intuitive.

It is just a idea expecting ownership.

So the only mechanism is that satisfy pareto efficiency, and strategic proofness and we call individual rationality, and individual rationality here means that each agent are,, must be getting on object, that is individually rational relative to what he owns.

He must not do worst than how he owns, meaning that the assignment must give him a house that is weakly preferable to house that initially own.

So either he could be, gets exactly how house he owns, or he getting a house that is better than what he owns.

Remarkably, just adding individual rationality achieves the goal of characterizing TTC, in addition to these two properties.

This is a result by Ma,

And there is a simple proof impact available I can show if you interested in.

And also TTC allocation is unique strong core, which coincides with unique competitive equailibrium.

This is very interesting paper by Roth and Postlewaite, strong core is exactly defined same way as before.

In other words, that robust against collision via weak domination.

Correponding to competitive, which very interesting actually, you can think of the TTC assignment as the competitive equailibrium outcome.

Give me a TTC outcome, I can come up with prices at which the outcome, assignment will be supported as some equailibrium.

Here is the easy thing.

So we know that the set of agent can be, actually the object for the matter can be partition into different subsets of that.

Subset's around objects, there are sign at different stage, steps of TTC algorithm.

The all one is said about objects that are assigned, to disguise of course,

In the first step, owe to, this set of object that are assigned in the second step and so on.

Imagine that the prices, imagine competitive market, well, there are prices for assign for each object.

And then, given the prices, I will show you that the demand equal supply, and everybody is maximazing the utility.

So and of course that, each person, what is the budget, budget is real environment.

Each person has one house, owns one house.

I against the following, any, for all the object , there are signed the first step.

You are signed the price, same price, let's say P1.

[70:00]

For all the houses that are assigned in the second step, you assign some other prices, the same price, P2, which is less.

Now put yourself in the shoes of the agent.

In N1, who owns houses, who ones houses in O1, end up getting with some houses in O1, need not be the same.

You have own house, whose market price is P1,

Now, any houses in O1, you can afford to buy.

Because, you sell your house, at the price P1, any house in O1, you can buy.

In for that matter, all the other house are cheeper, so you can buy any houses you like.

Obviously, you like your, I think that, this price get most sufficiently small, no, actually

One thing that is important is that either, so there are two ways to handle one possible problem.

The problem is this so, you sell your house, and then buy very cheap house, that's not very preferable to you.

But then you make so much money, you consume money.

So therefore, in fact, you may not end up with getting the TTC assignment as the competitive equilibrium.

There are two fixes, one is that in fact, you don't value money by itself.

So that's one assumption.

There is no other group in this world, and a money is not intrinsically valuable.

Or alternatively, as long as there are strict preferences, and carinal utilities are sufficiently different between the different object you rank differently,

If the price is sufficiently close to each other then also you can get rid of this outcome.

The point here is the, let's take the first view, under which according to which that you don't value money by itself.

That is quite clear, that since you can choose any group in this economy, you will end up purchasing, you will end up demanding, the one that's most preferable to you.

You can afford to pay by that.

Because, what we own is an option to evaluable also.

You get P1, and then you spend P1 to get the most preferable object.

And then so there is unit demend for the object, there is unit supply for that object.

There is no exceed man, because each person points to only one house, one agent who owns house.

And nobody else can afford to pay it that much.

There is unit demand and unit supply and also market clears

And for the other one, you can argue exact same way, to say that you know, you have competitive equilibrium in this market

in which object assigned the same sort of object as the TTC assignment.

You can think of this agent outcome more competitive equilibrium which could have another way to argue

that this is outcome pareto efficient by the first welfare theorem.

So let me take a brief break, three minute break.