Title: 시장설계이론1, 계약을 포함하는 매칭 (1)

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- ✓ **Dictated: 강은경, 강성호, 김신희, 김종백, 신원대, 현소형**

[00:00]

So let me just begin with a little bit of review of what we discussed last time.

So the general goal here is to have a fairly general model of a many to one matching where not only the parties, agents on both side, on this two different size matched to each other, in the process also contracts can be adjusted. Okay?

Adjusted in a way that at the end of the day we used to have a stable allocation.

So the matching is stable, also the contract that they signed is stable in the sense that you cannot find a coalition. Okay?

That can come up with better contracts for themselves. Okay?

So it's important to remind yourself, first of all, what the general unit of analysis

So here the objects that we focus on in general here are this X here, right?

X is a fini set of contracts, each of which has a three components.

So it has a description of who the part is are first of all

So for the first component, contract has this form which has three components.

First of all, the first component indicates who the doctor signing the contract is.

The second component indicates who the hospital signing the contract is.

The third component could be contract term which could be in term has several toppers in it, right?

They maybe could specify wage, you could specify fringe benefit, it can specify work hours and so forth. Okay?

So that's the object that we are deciding to, thinking about deciding.

But the several possible contracts are already given.

It's not like that they can come over the new contract. Okay?

But you can sort of imagine that this X is fairly rich set has lot of elements in it, lot of possibilities already exhaustibly accomodated in it.

So that it's actually pretty sufficient for them to find a contract within that rich set. Okay?

So here is the general assumption just to go back a few slides that we discussed last time.

So this is many to one matching model, so each doctor can sign only one contract which some are more than just many to one matching, right?

So it's not just one hospital that a doctor can sign, he can sign with a given hospital on one contract. Okay?

Again, this is without loss of generality like I said before, right?

Because you could talk about contracting to work for afternoon and contracting to work for morning.

You don't need two contracts to work for morning and afternoon, right?

You can sign another contract saying that I work for morning and afternoon. Okay?

So as long as the contract space is sufficient to enrich, there is no loss. Okay?

And then we assume that the doctor, each doctor has strict preference, strict rankings, preference rankings over contracts, which is relevant for him.

Namely that all the contract that name that doctor in the first component of the contract.

All the other contracts are completely of no concern to him. Right?

Because all the other doctors are for other doctors. Okay?

And then hospitals also likewise can … now the difference is that hospital can sign contracts with multiple doctors. Okay? One for each doctor.

But for each doctor, it cannot sign more than one contract for the same reason. Okay?

But can sign with multiple doctors. One contract for each. Okay?

And the most importantly, hospitals actually have strict preferences over a set of contracts. Okay?

Set of contracts are naming it, naming that hospital that is relevant. Okay?

So it's the same as before except now what it chooses, the alternatives are the matters for hospital.

[05:00]

It's a set of contracts instead of just one contract. Okay?

So the preference of doctor is easy, I mean, just as before, but we are going to still use this same formalism, namely the choice function.

So choice function is a way of describing preferences in general. Okay?

So choice function gives you given a set of available contracts. Okay?

It indicates what's the optimal contract or optimal set of contracts in case of hospital. Okay?

So for doctor given X prime, CDIO X prime, gives the best contract for that doctor. Okay?

So in case there are no contracts of course that name him, the optimal contract to ignore(05:59) contract because none of them optimal.

He is now allowed to choose a contract that doesn't specify his name. Okay?

And CD of S prime, so D is the set of entire doctors, is a union of optimal contracts for all doctors.

This simply means a set of contracts, each of which is optimal for some doctors given the set X prime of contracts available for them jointly. Okay?

RD is just contracts of rejected that are not optimal for any doctors. Okay?

So the order of taking this operation matters as I said before.

For hospital side, the same rule applies, namely that a given X prime, given the set X prime of available contracts are what it chooses is a subset of this set that is optimal among all subsets of X prime.

Name again only for those contracts that have the name of the hospital in it. Okay?

Again, if X prime does not include any contract involving that hospital, then it has the optimal choice as null set. Okay?

Or none of them are acceptable of course.

That's another possibility where the optimal contract could be a null set.

Again we use the same notation.

The crucial notion of stability here is, just as I remind you, involves true element just as before.

One is the requirement that the contract is individually rational, which means not necessarily comparison relative to null contract.

It's a little bit different in a general setup, just as before namely that for doctor, I mean again relavant comparison with a null contract, not being able to work, not working for any hospital has to beat that namely that X prime is a stable.

So allocation is a subset of X. Okay?

First of all, allocation is a subset of X.

And the allocation is stable, if it is individually rational in the sense that given X prime.

So for any contract involving a particular doctor must be optimal for that doctor given that the doctor has choice between either choosing the contract or no contract. Okay?

For given hospital, any set of contracts involving that hospital must be optimal if that set of contract is available to that hospital in the sense that the hospital should have no incentive.

Starting from the set of contracts, the hospital should have no incentive to drop any of the contracts which could mean, you know, dropping some doctors along with some contracts.

Okay, that's one condition.

The second condition is just general blocking here.

So there should not be alternative set of contracts differing from that X prime itself actually.

So there should not be any hospital H. Okay?

And a set of contracts differing from that, so each of this contract here should name that hospital. Okay?

[10:00]

But different from the set of optimal contracts that are relevant for that hospital. Okay?

First of all, such that those contracts, this alternative set of contracts all involving hospital H should be also optimal, should be acceptable to the set of doctors the contracts involve. Okay?

Set of doctors involved with a contract.

So doctors must find those alternative contract in a sense better, at least weakly better than the contracts from X prime, contract X prime,

meaning that either the alternative set of contract retains the same contract that used to be optimal on the X prime for a given doctor in here.

Or it suggests a new contract but that domintates the old contract. Okay?

That's what this subset inclusion means.

So X double primes are acceptable to the doctors that are involved in those contracts.

Okay, that means that actually those … actually all the doctors, so the hospital H employs even before. Okay?

Because it's CD, right?

And then the new contract is actually optimal when the doctor has all the contracts which X prime as well as new contract available.

Simply put, there is no alternative set of contracts that the hospital, a given hospital as well as those doctors that are associated with those contracts should jointly find profitable to defect to(12:23)

There should not be such set of contracts. Okay?

And here is our restriction on preferences.

So we started out with general preferences. Okay?

General, given by this C function. Right? Choice function.

Now we are restricting the set of contracts and restricting the set of preferences in this sense.

So we are moving from small set to a larger set. Okay?

Small set to a larger set that includes ... larger in the sense of set inclusion, not just in terms of size or cardinality is also in terms of set inclusion.

When we do that, what used to be non-optimal contract, when you have a smaller set available to you, should not be also optimal when you have a larger set of contracts. Okay?

It sounds as safe(13:24) that this is not a restriction at all.

This sounds as if that this is just follows from complete ordering.

But that's not the case, that's the actually important thing.

So again just to give you an example, so say that there are …

Doctor 1. Forget about wages or contract terms, just a simple matching.

The total set is just this. Okay?

Let's focus on a given hospital and let's say that … so CD was …

So here is one possibility, first of all. Okay?

Then, let's say CD1.

So I guess that … let me just do it this way and then change of course.

Here, when you make this statement that is no real restriction on preferences involved. Okay?

Because D1 was the optimal choice, choosing one doctor was an optimal choice when both doctors are available, when this irrelavant choice is unavailable, meaning that the inferior choice unavailable.

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Now he continues to like D1, so that is no restriction on preferences involved here.

So here in this case, this is also consistent with this assumption, right?

Because RD, smaller set, is nothing, right?

You are rejecting nobody, so this is the same as this RD ... is what ... D what. Okay?

So in terms of set inclusion, this is true.

So this is consistent with this statement here, this condition.

So far, this is not a result of any sort of restriction on preferences

But the following, the next scenario would involve restriction that is that … suppose that this is in fact the case. Okay?

So in fact when you are able to choose both doctors, in fact that turns out to be … choosing both doctors is optimal for you. Okay?

Now, what is this? Okay?

The warp, you know, the weak action of revealed preferences or the fact that the hospital is complete ordering, preference ranking of a subset. Okay?

Have no bite here. Doesn't say anything.

Here, warp suggests in this case that this has to be the case.

Because this is also optimal choice here when there are more choices available when you remove inferior choices that the optimal choice still remains the optimal choice

Here, the same logic doesn't apply because when everything is available, the optimal choice was to choose both. Okay?

When you have one choice available, you know, again what used to be amidst true that the set of choices shrink here when you move from here to here. Okay?

The set of choices shrink from ... the set of choices here was what?

Choosing nobody, choosing only D1, choosing only D2. Okay?

It's important to know what the alternatives are.

There are four alternatives.

And we found that to be optimal. Okay?

Now here, set of choices is just this, right? Okay?

Now, what used to be optimal is no longer available, so warp has nothing to say here about what the choice should be. Okay?

However what this says, I mean, we are not yet clear because … but what I am saying that this will be equivalent to, in this context, will be still this. Okay?

Even when something that is part of optimal choice is unavailable. Okay?

The part that used to be optimal, the doctor that used to be part of optimal choice, should still continue to be optimal when you have a smaller set.

And so ice(18:35) that relevant, the same thing here, what is RD in this case?

It is nothing, and what is RD1?

It is nothing, right?

So therefore, this is a subset of that. Okay?

So that is not implied by what? Okay?

It looks deceiving.

It looks like, as if that there is no real binding condition, restriction involved on the preferences, but there is restriction. Okay?

And then at this point, actually we are going to sort of construct lattice(19:31)

So lattice is a pair of set. Right?

Pair of some ground set and an order on a binary relation. Right?

We are going to think about a lattice and …

And, the lettice we are gonna consider is power set of X ok?

[20:00]

So, another way is that the set of all subsets of contracts

That's the ground set

The older given by, older is given by the set inclusion

So, the way did I , did I say anything here , not here

maybe in the next, here , yeah , here

so we are gonna bulid a lettice the ground set is all sort of contracts

no, it's a set of subsets of , all sort of subset of contract set

so power set right?

And older is given by set inclusion , relation ok?

so in particular, if any to subset , which is an element, two elements of this

power set

where one is a super set of the other, X' is super set of X

we say that X' dominate X according to binary relation

and then, one thing that noticed that is going to be exploded , it is assumptionthat this set is monotonic ok? In terms of set inclusion

so if you one set dominates the other set ok?

that set of reject, the set of reject essentially also sort of monotonic relation ok?

so if you find a , we don't choose a set of contract when you have smaller set

available

also you choose not to , we don't choose those set contract when you have

bigger set including smaller set available

and then, we are gonna consider the mapping , it's a set valued funtion so

start with, start this mapping defined from the power set of X to itself ok?

So, the formal fame , approach is here somewhat different hatfiled milgram

so I find that to be necessarily complicated , this is actually more clear cut and as acatually, as it precises , is what you know there is no real loss in doing this way

because they have actually product right? Prepare of this is ,ah , cartisan product of X by X

but , in fact, there is a mistake that you should be , domain should be power set rather than it's a set X ok?

that was the, treatment in the paper but that was a mistake

so here is the mapping , so given Y which is set of contracts ok?

mapping is the fine in this way ok?

so minus sign here , simply mean set minus ok?

the usual set minus ,ah, simbol ok?

so starting with a contract, set of contracts available to hospitals collection of hospitals

fisrt of all, subtract of those contracts that are rejected from this set ok?

and then in turn, those rejected sets are subtract from, I mean, so first consider the set sort of rejected , in the substract of those sets from the entire set of contract ok?

those are now available to doctors and then substract of rejected from those available set

and then, you enter, you get this final set

so the set valued mapping

the important observation here is that this mapping is monotonic ok?

you can easily varify by doing this right?

so consider this or in fact this , these are same thing

do you get ok? That's what you need to show

intuitively it's quite clear if increse Y, ok? Expand Y can make it bigger

ah, RH is monotonic ok?

we are subtracting more ok?

so X minus this thing here goes down ok? That shrinks

with that's the shrinks , one thing that I notice last time but not today is this is also monotonic ok?

[25:00]

So that's that shrink ok? So if you are setting subtracting of all smaller set , the whole thing actually gets bigger ok?

Um, so two minus make it positive,Y plus , so that's exactly the point ok?

So, um, why is this monotonic as well? Given this doctor's special form of substitute preference , that's the point

The fact that doctor has completed order , so doctor can choose only one contract and the fact that doctor has a linear order of all contracts ok?

that means that ah, RD is also monotonic ok?

because if we are given more contracts , you choose only at most one contract ok?

and then you reject everything else ok?

its just that , um, ok.. And then by the task is fixed point theorem , we know that when

we are given , first of all,

lettice which is partially order set consisting of ground set of binary relation with a lettice requirment that we talked about last time

ah, we are given lettice ,and if you have a mapping from lettice to itself ok?

which is isotone, which is a fancy way of , you know, saying that is monotonic ok?

ah, then it has a fixed point, fixed point is well-defined , also set of fixed point

itself it's a complete lettice ok?

so final lettice is complete lettice

ah, ok. So then we need to establish three things, that's one thing

the other thing is how we can find the so the fact that is complete lettice means that also there is largest element and the smallest element in the fixed point

that's just you think ok?

that's existance of things

so often you don’t have it ok?

because we are talking a partially order set after all

so some of them , elements are not comparable in the beginning.

the fact that it's not a , it's not a what you going to think, right?

it's not a something taken for granted , the fact that , largest element and lowest element , small element exists

set of fixed point not empty lettice ok?

and then you can find the largest element by starting from the largest element

of original lettice and then, keep applying the same operator , and it will go down and then it converse it in $\oint(Y)$

because of the underlined set $\oint(Y)$ to a largest fixed point ok?

we start from the lowest element of this guy ok?

lowest in accorting to this $\oint(Y)$ relation ship of course

and then keep applying the same operator ok? Which is monotonic , then you will reach a smallest element of the fixed point ok?

Ok. So , if contracts substitute this , since P is isotone under the standard inclusion order RD is isotone , therefore, P is isotone, um, and so therefore , three things that we apply here , um, and let's say , H is fixed point of P ok?

So meaning that ok?

ok?

and then let's define XD to be X - RH(XH)

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if that's , you define this way remember what if Pf XD is right?

ah, it's what, X minus RD of X- Rh of XH

That's how we define the fact that XH is a fixed point means that this is ture ok?

we decided to call this guy XD , so therefore, we can get the pair of equation

this is one, the second is exactly this ok?

and here I mean now it's time sort of to go back and sort of think about economics ok?

so far, it is mathmatics , it's time to think about economics

what is the economics ? What is interpretation of XH and XD

we can think of XH as this sort of opportunity set , I mean, that was it , just like the budget set when you talk about utility maximation problem right?

the sort of contract are available for hospital's truth from ok?

likewise, ah, likewise , XD interpretates , you can interpretate XD , as the sort of contract that are available to the doctors to choose from ok?

so essentially ah, you can , will not say these opportunity set , I will make it clearer ok?

ah, if you interpretate this side to call that in that way, essentially ,ah, the, you can think of entire contract ok?

what it says is that entire set of contract can be seen as a million of this two set ok?

this is XH and this is XD ok?

so it's nothing that belong to either of them seriously

ok, that's one implication of F(XH) and XD being the object of define sort of fixed points

so XH is the fixed point XD is defining this way of course

ah, so what the fixed point equation is, there is other two point equations ok?

what the imply is the following ,the set of contract XD you decieded to interpretate as those contract sort of available to hospital

I exactly those contracts ah, that are available after we take out , after we take out contracts that , contracts doctors reject ok?
when they have XD available , so again , ah interpretate XH as the set of contracts available hospitals , opportunity set for hospitals , this is opportunity set for doctors ok?
opportunity set , what is available to hospital for them to choose from is precisely those contracts ok?

ah, precisely those from the initial set of contracts that are not real rejected ,

that are not rejected by the doctors ok?

as of optimal when they have the opportunity set available to them ok?

and likewise, the set of set XD of contract of what is available for doctor's choose from , precisely those contracts that are not rejected by hospitals when hospitals choose from XH ok?

that's sort of what's going on ok? That's what the fixed point,ah, implies ok?

and this feature here , it's very nice feature because ah, essentially , this is precisely what it will give us the next result

the next results are basically saying that if XH is fixed point that the intersection of this opportunity sets ok?

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it's precisely stable allocation ok?

so and the converse is also , I mean, also this cut-up here , the converse also holds , I mean, namely that ok, ah, set X' is stable set , if that, only if ok?

it's an intersection of these two ok?

define either fixed point equation

another is that essentially the fixed point is equilibrium to, I mean, it's not

fixed point identied as XH

but you know in some sense , intersection given by the fixed point characterized entire set of , set of ,um, a stable allocations ok?

so that could be mutiple stable allocations ok?

XH that is ah, fixed point that could be mutiple fixed point ok? They could be

multiple fixed points

ah, so therefore, there could be multiple stable allocations ok?

any stable allocation must be defined by the most correspond to a fixed point

of that operator

I say correspond to because this fixed point identies this set allocation , stable allocation given by intersection

so we are gonna prove that , so first of all, we are gonna prove that if there is

a fixed , if you get given a fixed point of that operator ok?

the intersection you get , which we define to be X' ok?

we are gonna show that that's a stable allocation

next part is the opposite namely that if it's a stable allocation , it must

correspond to some fixed point ok?

so first of all, this is simple algebra , but it's also quite clear ok?

um, just looking at this diagram essentially

ah, if you looking at the intersection of course , I mean, it's not yet clear ,

intersection is not empty at this point

ah, but, it will be because so what is intersection ah, intersection is that first

of all, ok., so exactly what's intersection we decide to call X' , ok , but looking

at quite clear that this X to be CD of XD

when the doctors are given this , ok? What is the interpretation of this ok?

That's the reject ok?

ok? The entire set? that's the result ok?

and you can also varify this algebra as well. XH is given by, actually, in this

way given by, right, by fixed point equation , XH is given by this. This is from

following fixed point equation

this is what, result of taking out from XD ok? Those sets are rejected by

doctors ok?

so therefore, what's remaining must be the optimal choice of doctors

likewise, it is also equal to XH ok?

right, so, for instance , this,ah, also can be seen as the intersection of XD and whenever it is not rejected by the hospitals ok?

so and from here, now, this is one thing that we note ok? By sort of argebra

simply first of all

but then we can deduce from this individually rationaltive X' ok?

Individual rationality of X' why because this wil imply that this is true.

So suppose that you are given Xd and you choose X' collectively.

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You are just given X' and will continue to choose X'

Here, are we using substitute preference run out?

then since you are going not even using that

You just reveal your preference.

Again the argues to be the optimal choice which is X' one is available to you and that's only thing availiable to you, you choose it

You will not choose any other subset which is subset of X' which was availiable here.

Not chosen and it is not be chosen either.

Likewise this, even without using substitutability of preferences for the hospital side.

So, what's you get back, we have established individual rationality.

So on the hospital side, there is no incentive to drop any contracts.

Now, next, I am going to think about some alternative set of contracts, that involved some hospital rates

Take any hospital H, a next lebel prime, named that hospital.

And suppose that this is available, this is acceptable for the doctors, so meaning in the sense

Because, for this to be alternative sex that can possibly block the original set of contracts

This must be one of the conditions that needed to be satisfied.

So that's you can restrict the tension to those contracts for satisfying this requirement.

Think about here in the context, this is X' set.

And we are talking about particular hospital and let's say that among the set, there is some subtitute contracts that only satisfiy the particular hospital.

And that's still available to all the doctors.

Actually, let's call it D', D's are doctors that, it doesn't really matter, doctors that are involved in this new contracts as well as all the contracts.

There are signing the H [?42:44 on the old and new]

That's all substitutable doctors and for those doctores they have these contracts, having to do with those for any doctor signing with H in the original contracts, they are here.

Those contracts are all here.

Now, we have talking about alternative contracts that are acceptable to them, meaning that either they include the old contracts.

So X' specifies for some doctors in here.

The same contracts X'

X'' can overiap the X' meaning that it may repeat the same contract, they are same.

For those doctors you know that, they have no more choices.

They just have old choice, they still continue to choose them,

For other doctors, it may involve new contracts.

Actually, it may involve some new doctors which used to sign with the different hospital.

So any contracts, the sign, new contract sign with H, this involved.

So those doctors may be doctors who signed with different hospital before and X'

But those doctors are included in any case, so in fact that you have to include all the contracts as well.

This contract includes, all the contracts that involved H, either on the X'or X''.

So includes the entire set of doctors, who sign every dates, either old or new contracts.

So for those doctors, the X' may involve new contract, some new contract.

Now they can, they have, and their choice said, those new contracts as well as old contracts.

When they entire all contracts are available, when they are presented the new contract, what it sells with the new contract must be optimal for them.

[45:00]

But the means is that particular new contract, whatever that maybe cannot include any thing in the reject set.

So, reject set that included them essentially.

It could include here, some contracts in here, but you cannot include any contract in here.

Why because of this X', of course, if you are given a new set of contract, as available set of contract, which does not include X'.

But of course that you know, you may choose something in here, you may find some contract in here as optimal.

If you have to choose not, I mean, this contracts are not available to you.

Remember, by the reveal preference, whenver all those contracts available here, you continue to reject all the contracts in here.

So, therefore, what this means is that, X' should not have any overlap of, with, those set that I rejected in the past.

Because, X' is available to them.

So what the means is that X'' whatever the means, that is, must be overhere.

So what is Xh? Xh is exactly, the entire set here

So in other words that, whatever the new alternative sets for them to available to the doctors,

that concern them, doctors they are involved in this contract cannot include any of those sets that I rejected

So therefore, all those new contracts must be within the set,

But then if they are all the case, clearly, CH hostpital, XH, okay, must not be equal to XH,

So, what I am trying to say is that so let me see,

This is follows from here, so X'' is also available,

And since double prime is different from that guy, so this is different from X''.

Okay so therefore, what we have concluded , let's go back, what we require for set of contracts to block, this, okay?

So if we block what we have shown is that if you consider any alternative sets that is redundant set, but acceptable to doctors, it cannot be the optimal choice.

Okay?, for the hospital

So in other words, you cannot possibly find the alternative set of contracts that are attractive for jointly for all the hospitals and all the doctors that are involved.

Let me just start do one more

So, the other side, the opposite side, to get that, suppose that X' is a stable allocation.

Now we partition X into 3 different sets.

So now, forget about this fix pointed equation.

So we start with assumtion that X' is stable allocation which means that we don't know yet if X' can be explained, identified by the, satisfied the fix point equation.

The point that is, you want to have the goal to actually show that it must satisfy the fix point equation.

which we had concluded, complete our characterizatoin that any stable allocation can be completely characterized by fix point equation.

[50:00]

So we are going to start with arbitrary stable allocation and then end of day, we will be going to show that it must be satisfied fix point equation.

Again, the goal here is to show that this stable allocation, the set of stable allocations can be completely characterized by the fix point requirment, they coincide the fix point in a sense

So start with arbitrary stable allocation, you can then partition the entire set of contracts into 3 sets [?50:35] which is X'

What is Xh' while this other set of contracts, the doctor strictly prefer to X'

So for any doc.., so for X', includes set of doctors,

Consider each and everyone of them, for each doctor you ask what's better contract for him ?

This is collection of all the contracts that are better, for each of the doctor that are involved here

Now, this is the set of contracts that, are worse than X', strictly worse for doctors.

Again the same, look at every doctor here, and ask yourself what other set of contracts are dominated by the contracts he gets on the X'

Collect them here, you can partition them into 3 sets in this way, and then, we can then think about this essentially this equation, this relation here.

If you just take the union of X' and X'd, another set of contracts weakly dominated by X' for doctors

If you take the Rd of them, collect the set of, if doctors are given the set of contracts, what are they going to reject?

By definition, this is exactly that.

If we define, this guy Xd to be union of that, define the result just make it clear, then we can rewrite

So start with, we start it out with some set X', and you ask, one of set of contracts, that are strictly worse, and that's Xd.

What other some set of contracts that are strictly better

Now, and then what he is established is this, by definition, here is Xd', such as given by definition.

And then, we decided to call this a Xd.

Now, what, and then decided to call Xhd weakly prefer set.

So we have this set, and that set, we have this set, so they're just looking at it.

That's clear, this is X-Xh, right?

That's all that is, for the first, we got one equation.

This was we got equation, sort of trivially,

And next equation, sort of less trivial, that is also true.

So there, but I use stability actually, we haven't use stability at, right?

So we have to use it somewhere.

Here is the way, suppose hospitals are given these contracts.

So remember, what these contracts are, so these are set of contracts that the doctors the way they define them,

[55:00]

These are the set of contracts that doctors strictly prefer, to those contracts in here for them,

If hospitals are given, as an opportunity said as an available set of choices, these, union of this two.

Now, the way define the set, this is a set, that is strictly preferred by doctors to that set.

Now, my claim is the this hospital possibility.

Hospitals must reject all those sets the doctors strictly prefer to X'.

Or else, we don't need to trouble with stablility.

Why, because, suppose that, here, let's say this set here includes.

Let's include some part of Xh'

There is some contract here, that is not included over there

What is in here, included here but not included in here.

So I am going to show, say the, actually, so I am going to proof first of all this is the case

Everything in here, must be the cell of contract must be those of contracts that are rejected

when the hospitals have this set of contracts available to them.

Because what else, suppose the contract like that, that is not rejected, but the doctors actually react truth

Then those contracts are also strictly preferred by the doctors

For these other contracts that hospitals choose and then those are also strictly preffered by doctors as well.

And therefore, no formal blocking collision.

Those contracts, those contracts can be used to general X''.

Those are joint preferable by the particular hospital in here, those things are availiable to them.

So you can general X'' which includes everything and includes this hospital, plus, whatever that, let's call alpha here,

And that's strictly preferable, the fact that is not rejected, means it is strictly preferable over those set, that set.

And, actually, I am not quite precise here, because,,

The fact that some of this, some contracts here is not rejected, means that it will be part of optimal choice.

So need not necessarily, so [?58:41 only???] implies is that exists X'', that contains a contract in a Xh'

Such that, Xh is equal to Ch, for some hospital.

And those are clearly also acceptable to the doctors because anything here are strictly preferred to any contract here.

So, I the idea is simply is that we can find a set of contracts that can be used to block original contracts. Okay?

So, that's one side. What about, we are going to also prove that. Okay?

Why? Because, suppose that, we are rejecting more than that. Okay?

We are rejecting more than that. But that means is that X prime is not indivisually rational for the hospitals.

Hospitals will find this strictly prefferable to job some of the contracts, when this is given to them. Okay?

So, therefore, you get something like that. Right.

Okay?

So, what here shown is this and according to our naming here. Okay? Our notation okay? What is this?

That's just a Xh and this is what? This is Xd. Just look at the diagram there. X-Xd. Okay?

And therefore, Xd equals X - Rh of Xh. Okay? Again, so we have arrived at fixed point equation.

So, but, I guess that over the 2 directions the formal direction is more important. Okay?

Because, remember what really care about is the existence of stable allocations. And this formal direction establishes that

I mean that we know already the fixed point is not empty. And what it shows is that if you have a now fixed point then you can find the stable allocations.

allocation, X prime which is defined as the intersection of Xh and Xd. And what this direction has shown is that X prime is stable.

So, therefore, they exist stable allocation. And more interestingly we can.. So, let me stop here. What I am going to show you next is

to use these other two implications of the Tarski's fix point theorem, namely that you can find not only Tarski's fix point exist,

there is the largest point and the smallest point. And those can be found by running this monotonic sequence.

running this operator. Repeatedly either starting from the highest largest point in the lattice, and the smallest point in the lattice. Okay?

We will show you then how those operation correspond to a generalized version of deffered acceptance algorithm.

How we can sort of map that process into deffered acceptance algorithm. So, this is kind of nice because you have a sort of a nice math theory,

and then nice math has an analog in the familiar algorithm. Okay? So, let me stop here, and then take three minutes break, and come back here.