Title: 시장설계이론1, 다대일 매칭 (2)

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- ✓ **Dictated: 강은경, 강성호, 김신희, 김종백, 신원대, 현소형**

[00:00]

Let's bigin. So, I was thinking a little bit about what to do in terms of the requirements. I have decided now that in addition to having I guess three problem sets. You know there's one already, and then there will be another one not today.

Because I think that the second one will basically cover the material that will be discussed.

Starting this Friday, so, I will assign the second one tomorrow, not tomorrow on Friday. And then there will be one probably at the last class.

Okay? I mean Friday a week from this Friday. In addition to those three problem sets, I'm gonna probably have an exam. In class exam just to I guess give you more incentives to review the material that you have discussed.

Because, without really repeatedly going through the material and trying to understand, you don't get much out of there on the class. Okay?

So, that's a simple fact. So, I think that I'm planning to give you the opportunity to get yourself motivated basically to I mean because the material here, some of material, some of it takes a while.

And really a repeated inspection to understand better and better. So, I think it's a good thing for me and everybody to help each other

in terms of creating more chance.

And incentives to go over this materials. So, it's going to be reasonable in the sense that you know not going to be totally new problems in some sense.

But you know problems that a kind of related to the problem sets and the class material. So, and then I may actually have another requirement which is to do a little bit of referee report or you know review of papers.

So, I'm gonna set of offer you list of papers and so, I think this is also a good exercise to work through research paper in this field.

Number 1 to give you some flavor about what kind of research you can do.

And second of all just to understand better about themethodology So, how the methodology that we have learned here can be sort of evolved and improved and extended to deal with other problems as well.

So, I think that hoping to give you a better opportunity to for it to get something more out of this class. Okay?

So, what did we do last class, well, we extended frame work from one to one matching to many to one matching settings. Okay?

In a sort of, we did in a sort of simple baby step in the sense that you know we made a very strong assumptionon the part of the

I mean the critical issue when you extend and move

from one to one matching to many to one matching is to, is the preferences on the part of the schools.

At the side which admit matches with multiple agents on the other side. So the school side, the college side is that side.

And the question is really what kind of preferences that we can allow.

Okay? For the colleges and as a simple baby step what we did was to assume that this college's preferences are responsive by which I mean, we mean that there is a sort of well defined sense of rankings.

On the part of schools about students. So, and the preferences of different schools a sort of represented by this rankings.

And that means that if I compare between the two, my preference between you two, that's not depend upon who else I get basically. Okay?

And that turns out to very useful restriction, useful in the sense that everything that we've learned from one to one matching kind of our carried over very easily to many to one matching settings.

So, all we needed was to come up with some sort of an isomorphism. So, recognized mapping between a way to interpret many to one matching as so special case one to one matching.

All we needed was that be decided to interpret each positions that are available for the college side as a separated entity, separated individuals if you will.

[05:00]

And then to have well defined marriage problem, we needed to define preferences. And for each position on the college side,

it's fairly easy because you know we can endow which position on the same preferences as the college itself.

Okay? And then for the student side there's an issue about you know how they rank different positions within the same school.

Now, we decide to, without arbitrary, in the sense that it's a sort of assigned some serial number for positions.

And then endow preference endow which stuent preferences is a bit which rank basically a position with a lower number,

lower index than the position with a high index.

Okay? As it turns out then with that we have established although without proof. The fact that you know matching you know college admission problem is stable if and only if the corresponding matching. Okay?

The related marriage market is stable. With that sort of a conversion if you will that interpretation we are able to basically recyle a lot of results. Okay?

First of all existence of a stable matching come out free, fairly easily because instead of looking at the college problem, we look at the associated related marriage problem.

And then we can learn a differed acceptance algorithm to find the matching which turns out the stable ,of course in the related marriage market problem.

But, then we know that there is a way to move back to a matching in the college market which is also stable. Okay?

So, therefore you are able to find the stable matching. You are able to also find out that this differed acceptance algorithm finds a matching

that is extremal in the senses optimal for each side for the proposure side.

Okay? If you learn a student optimal differed acceptance, you'll find, if you learn the student proposing differed acceptance, you'll find the student optimal stable matching.

If you learn the college proposing differed acceptance, you'll find the college optimal stable matching. Okay?

And then the same argument that we used to prove one sided strategy proofness. Okay? For the student side. Okay?

also carries over because all we needed is to start with any college's admissions problem.

Now, look at the associated related marriage market problem. Okay?

And then learn a differed acceptance student proposing differed acceptance.

And apply the theorem that we established for one sided strategy proofness for the student side to find and to conclude that mechanism basically strategy proof for the student side. Okay?

meaning that it's a dominant strategy for each student to submit truth for rankings of colleges. Okay?

turns out however the same cannot be set about college proposing differed acceptance algorithm.

So, we showed an example which illustrated basically. The fact that there is, there can be a situation where the college may, there is college I may want to lie about the ranking.

Depite the fact that everybody else sort of report truthfully. Okay?

So, the one sided strategy proofness extends but only for the student side not for the college side. Okay?

And the same example that we used also shows that the weak pareto optimality result.

That's not hold also for the college side ok?

It continues to hold for the student side

Because therom we established before applies there

But for the college side does not ok?

Again the same example basically illustrate that

You should be able to even construct example because example is very

simple

Sort of modification , this is ah, fairness matching type example ok?

Now, so then, at the end of the last class, what we did was to see if we can

relax this reponsiveness assumption on the side of colleges ok?

Can you accommodate broad class of preferences on the side of colleges? Ok?

And, general way to do it is by way of using so called choice function ok?

Choice funtion does what, it's a mapping from any set of elements ok?

Into its subset ok?

[10:00]

It is initially , should be choice correspondent , it could be mutiple subsets in

case, I mean, let me just define what it is

It maps from any given set into its subset with a interpretation that the chosen is optimal for a given party ok?

So, if you think about a choice funtion for C ,. Choice mapping for college C

The low case is , use the single agent ok?

Large case, capital C is often used to indicate the set of colleges ok?

For a given college, choice funtion for college C gives you subset of any given set S ∽(tilde)

Which turns out to dominate any other subset of that set S~

That could be mutiple subset of sort of optimal , in case the preference involved in differences

But we are gonna ,you know, extend , we are gonna use the same assumption for mainly that the colleges , each college has chic rankings over

all subsets ok?

So, complete, I mean, this chic preferences

So, we that assumption this becomes the function ok? For any given subset , there is unique subset of that

For given, any given set is unique subset of that it turns out to be optimal

So, this means that you can think of this as an opportunity set or woman you feel like budget set if you will

So, set of available choices in terms of subset. there is unique subset that is optimal ok?

It is basically what we mean by stick ranking.

[student speaking]

Ah, yes, that's right , this is a typo , very good observation

This must be S~ ok?

I didn't , ok.

Um, ok. And then ah, we are going to , as you trun out , the point that , it's going to be important here and also with a Hatfield milgram paper which we will turn to a llittle bit later

Um, so the frame of that we have , the old nice results including things like existance of stable matching or not

Ah, holds with some restriction on preferences we can do more than , we can do better than reponsiveness

But we cannot accommodate all possible types of preferences

So the types of preferences we can accommodate , ah, known as substitable preferences

So what does that mean that, that means that, for any set containing S and S'

If S was optimal, given this choice available element ok?

S continue to be optimal when you subtract another set , another element S'

Different from S of course ok?

If something was an optimal choice, part of optimal choice, so here let's subtract thing as an element of choice is not a, you know, this is an element of subset that is optimal given S ok?

Now,ah, it's going to be also optimal , if it subtracts other elements different from S ok?

So, there are two situations where this matters

Let's say, this is the set of available choices ok?

Now, given that this could be optimal right?

Some subset maybe optimal ok?

This could be your choice

So, actually we are gonna drop H here later on ok?

We are talking about some entity like college , we are talking about this college's preferences and this is optimal set ok?

There is an element here , some student turns out to be part of optimal choice for that college given this available set of students ok?

Now, there are two possiblilities relating to this statement.

one possibility is, we are substracting that set ok?

that element here outside the optimal choice

In that case, clearly ,ah, if this, ok, if every element , every subset of S that this guy dominates ok

So, after substracting this S' guy ok?

Every element , every subset sort of available ok?

Given S continues to be available when you S, let me see, when you include

S' as well ok?

[15:00]

So, therefore, when you subtract this, clearly this set will continue to be an

optimal choice ok?

You don't need any assumption for that

The only assumption we need to say that S continues to be a choice ,part of

optimal choice when you subtract an element like that , is the assumption that the colloge has the well-defined ranking even it is possibly weak rankings ok?

Over different subsets ok?

Basically, minimal assumption of rationality

So, we are not even invoking substitutability at that point

The real non-trivial assumption ,ah, kicks in

substitutability kicks in , when we substract because one possiblility here is S'

, could be part of this , part of S * ok?

Now, an element the student used to be part of optimal choice, choices

optimal set is no long available ok?

So, therefore, your optimal set might be different in the even include

something else actually right?

And yet , what it is saying is that , whatever used to be available before and

optimal whatever used to be optimal should be , still should continue to be

optimal ok?

So, this sort of captures the idea that the college views differenct students as substitable ok?

Now, there is a , there used to be another student that was valuable with that

student gone , the remaining optimal student , the optiaml student that re-like it before you continue to like him ok?

So, it does not allow , is that, let's say, suppose that i mean, this is not ture ok?

Of course, i mean, if K, the student that taking out is not part of optimal choice , then of course, this guy has to be part of it

The only way in which that this fails is when the guy we are taking out is part of the optimal choice ok?

Suppose that there are the case, So, when you had , when S' was available

you like this ok?

But now, with S' gone, we don't like S ok?

So, that's kind of the , you know, situation of complimentality of preferences ok?

Because the valuable having S , crucially depends upon being able to have

S' ok?

so I gave you this example with a baseball team recruiting players.

and so devaluar particular player, let's see, particular catcher depends on whether or not you can get paricular picture, okey?

without him, we don't, we don't value him.

Whereas with substitutional preferences, this can not be the case, okey?

so now there is less of competition.

so you must, you should like it.

(students speaking)

so that is that exactly maximum of middle reveal preference

when you take out as prime that is exactly what we call reveal preference

where is here, you need more than that.

that is an assumption of the substitutability, okey?

right.

so in this case, in this case, where the guy are subtracting, taking out.

It's not part of optimal choice, okey?

and remember what's your choosing from, you are choosing from the set of subsets of S, okey?

when you take out something like this, you are reducing the set of subsets, okey?

now, this guy is the optimal with a bigger set of choices, choice meaning subset, offset.

now, it's still available, okey?

so there is, you face less of competition, fewer competitions, clearly this is going to be the optimal, okey?

so in that case, we don't even, we want to even invoking substitute ability.

but this statement basically covers all possibility.

in particular, the possible case where as prime actuallly was used to be part of, part of S, S dot okey?

okey.

so here is an example where preferences are not responsible but substitutable, okey?

you can easily check it is substitutable.

[20:00]

because, let's say..

if all three available, this guy likes this, okey?

when only one of them is available, let's say, S^1 the guy still likes that, okey?

S^1

with S^2 gone, okey?

the guys could demands the remaining one.

so that's, you know, you can see that this condition holds

fails responsiveness because..

because this guy liked when he is forced choose between the two, okey?

he has to choose, he will choose S^2, okey?

right.

between S^1 and S^2, if he is forced to choose the only one of them, he'll choose S^2 with S^3 available, okey?

he actually likes S^1 better than S^2, okey?

set of relative preference, relative ranking of any two students change as a fuction, I mean S, the college admits the other students, other students, okey?

and then.. Since we're generalizing preferences.

and we're getting rid of this quota business, okey?

so in the past, so preferences were presented by these two things.

one is the rankgings of individuals with responsible preferences along with quotas, okey?

so quota is implicit to introduce preferences.

preferences suggesting basically that you don't want to, you don't want to admit more than this particular number of students, okey

that's a formal preferences.

here we got with of the quota business, okey?

but remember, this is more general.

you could, you could capture the idea of quotas with that clearly, right?

so and then, so but we need to have more general notion of stability, okey?

since we have extended a framer.

so a matching mu is defined as before, right?

so is blocked by, I mean, so matching is mapping from..

did we define..

so it's a mapping from, let's say, mapping from union of students and colleges, okey?

into itself, such that, you know, it's right, student is, you must have some consistency, right?

in the sense, student is assigned to some college, okey?

that college must be assigned to that student, okey?

and then, here we are not using the convention that you know, any there is no notion of vacancy essentially, right?

and for each college, you are giving him a subset of students essentially, okey?

for students, we are giving a single turn college as a mapping.

and for college side, we are giving a subset of students, okey?

so that's the basic definition of matching and matching mu is blocked by student S.

itf, I see, I see, this is not correct.

we has to be S, okey?

so if it's, I'm just already contradicting myself.

so we, we are actually for the student side, we are allow him to met.

so the outcome of matching to be himself or college, okey? Just as before.
so this should be S.

it is blocked by a student S if college that is so assigned is worse than not being assigned to any college, okey?

and it's also blocked by a college if the college strictly prefers to drop some of the students, okey?

from among the set of students that assign to that college by matching function mu, okey?

so I have matching function, let me give you an example.

this is a matching function.

as give him a subset S^1, S^2, S^3, okey?

[25:00]

if college C prefers this to S^1, S^2 meaning that we should to drop some student.

then we are saying that the matching is blocked by the college, okey?

it could be that this may be better than not having any student, okey?

so in the sense that this a little bit different from, so matching not be blocked by a college.
it's a little bit different from that matching being individually rational in some sense, okey?

so sort of difference, that is a bit of difference relative to the previous definition of individually rationality, okey?

so in other words that all it says that is the requirement.

this is simply requirement that they should be know preference on the part of college to drop some of the students from the assignment, okey?

and then here is a notion of no blocking pair requirement.

so is that notion, first of all of blocking by a pair.

block by unmatched pair in S and C, they are not currently matched each other.

such that S prefers that college over his current matchiung which could be matching being matched himself, okey?

and college prefering to replace, prefering to have S, okey?

either in addition to everybody is assigned to or instead of somebody, okey?

so remember we don't necessarily impose some quota in this case for college, okey?

so ku is possible but the college may want to additionally may want to admit as student as in addition to every student, all the student is already having ok?

that's one possibility what this means is that if you add this set is x to add this student to this set of available this set of student is currently assigned ok?

part of their planed choices which could occur different way like I said it could be that the college may actually add this student in addition to the choice

it could also be that it could take out one one more more than one even right?

students and then replace them with that student ok?

anything is possible

so that's a blocking and then now we are ready to talk about stability of matching mu stable

if it's not blocked by not block neither by any individual

no by any pair of student and college ok?

given substitutable preferences and then once be that here is the main thereom

so given substitutable preferences are stable matching exists ok?

so that's a generalization

it could be that this is possibility what I'm saying ok?

which is different from this right?

so this is another way to say yeah but this is enough

I mean if this is the case than this is still right? This is formal blocking ok?

you are blocking this existing matching by adding this student

you strictly prefer to do so so the college is on board with this idea of blocking

and student is also better off from the process we'll go along in this blocking ok?

stable matching exists and the uh proof this um so, you should look at proof I'm not going to be very carefull with the proof

although I'm going to give you the main idea

[30:00]

since some sense the Hatfield Milgram results substitue more of these ok?

but it's useful I thought to begin with the intermediate step here

ok so the idea is also a proof is also constuctive ok? so you are generalized the defferd acceptance algorithm

here is the college proposing defferd acceptance algorithm

you can also construct student proposing defferd acceptance algorithm as well it's very similar to what we had before

except in its stage, college propose to set of students

there is no notion of quota

so it could propose any subset of entire set of students

the most prefers set of students ok? And then those who are proposed the student who receive proposals

then individually reject ok?

I mean all but the most prefer acceptable college ok?

so each student has quota can not go to model one

college so must end up choosing only one at most one ok?

if no proposals are acceptable then there is no choice

if there are there is at least one proposal that is acceptable the college their student will accept it at lease tentatively ok?

for that ground.

there are multiple proposals you can choose at most one you can choose only one ok?

and he will choose the best according to his preference rankings ok?

and then let's move on to the next step ok?

so then I am a college I have made a number of proposals so the most prefer set of the students

some of them rejected mine ok what do I do? Ok?

I set of re revisit my situation ok?

now I have no choice to withdraw my proposals to those who have accepted ok?

so that's not my choice but I'm allowed to at more proposals to those are the students ok? Who have not yet rejected ok?

so I cannot revisit those students who are rejected means there is no elemental prominence

otherwise the mechanism never algorithm never converges

I mean something must be permanent ok?

the rejection must be permanent

so I cannot I'm not allowed to revisit those students who rejected me

but I can make additional proposals to those students who have not yet rejected me ok?

I cannot however touch the proposals that I have been accepted

I cannot recant those I cannot retract those offers ok?

that's basically what happens in the second stage on

and then those students who have received proposals

so every student also revisited his decision ok?

he has accepted it all before now he make additional offers ok?

those who has not received any further offers

he cannot of course reject what he accepted before

but he has multiple [?33:29] for more

then this student can, most true at least at most one ok?

ok it's so once you have an offer, you are guaranteed to be matched just as before if there are students ok?

you can as time goes do only better because you have an option you have a full back option deffered

you can never do worse than that

you are guranteed to have that as time goes you may get more better offers

because some are their colleges whose offers rejected by students may actually like your some points

and then make proposals to you ok?

and then you compared them and then choose the best one

as same element just before, same sort of general features inferred

the only thing is not trivial here

this kind of new is that is sort of requirement that the college cannot retract the offers that have been accepted ok?

so there is no fighting constraint I said workers I used go back and forth from works and student colleges you should strike out this and then the algorithm stops when there is no rejection

now the critical theme that you have to think about here for the colleges

so where does um so one think substability has a bite in one element of this algorithm

[35:00]

so in other words that this no fighting constraint that you are not able to you are not allowed to retract the offers or accept the ones ok?

so that's a requirement but the point here is that with the substitutable preferences that requirement is never binding ok?

you don't never you never wish to retract ok?

why is that? Ok?

so the college start allowing the large options you can choose any subset of the entire set of preferences in the set of students

as time goes the opportunity set if you will of each college shrinks ok?

the set of availiable students shrink as time goes,

now, so what used to be a part of optimal choice before, when the set that was availible, there was huge

remains optimal, right? as the set of student availible to you shrinks

each comes from the substitutability assumption

giving substitatubility assumption, even if you allow to retract your offer, you never wish to

so in other words, basically what is going on here, is that colleges allow to essentially, making the best choice,

when it makes a proposal to a number of students, including those proposals that you retain,

so there are one more to think about, the proposal you made before, they are not rejected so far

you can think of them as reproposaling first, supposed the college instead of requiring them to retain the first,

suppose the college is basically each colleges allow to repropose everybody start a fresh each step.

meaning that, even though who accept yours first, you repropose essentially.

you allow to retract

give youa substitute preference assumption, we are going to continue to repropose for those who have accepted.

so the set of student you proposed to at each step, is really true optimal set of students, among the set of students who have not yet rejected to you.

so do your best essentially.

and then, so what does that mean, so let me first of all make a toward the proof of this theorem.

let's sort of make a couple of observation.

there is no blocking by each agent, let's think about the student's side.

it's clear that no student will block the final matching that comes out of it.

because, at each step the student never is forced to accept unacceptable college.

always you know, it's acceptable college, whenever we get, accept some proposal, it's always acceptable college.

so there is no blocking by any students

at each step, and therefore, it's true in the final step.

there is no blocking by each college, either.

why because, like I said, at each point, each college is optimazing, within set of students, who have not yet rejected.

so therefore, set of students, so there is no incentive to drop some of the students from this.

so there is no blocking by any college.

and furthermore, there is no blocking by any pair of unmatched college and student.

why because, again, coming from the same observation, that is that

at each step, the college is proposing to the best, most prefered set of students, including those who have accepted him, that college before.

in counting one of them, as the part of the proposal, made a fresh each step.

each college is proposing to the most prefered set of students, among the set of students, who have not yet rejected that college.

so suppose that there is a student, that the college would like to have, which is not part of the final matching.

it must have in that the student have been approached by the college before, but the student rejected that college.

[40:00]

if there is such a student, satisfying this requirement, the student must have rejected that college, meaning that this cannot be the case.

so there is no blocking by unmatched pair.

you could have done the same proof would student proposing deffered acceptance algorithm

and so you can, you have to see that the importance of that assumption, that substitatubility.

whenever you see a statement about theorem that uses some assumption, you have to always ask yourself.

where this assumption is used in the proof

okay that is sort of more critical way of understanding theorem.

you could have done the same thing with a student proposing deffered acceptance

and here there each student makes a proposal, college then look at all the proposal he has received.

and then the mass make a best choice

as time goes, the college made rejects some students,

so it's frist step, students propose the colleges, colleges basically look at the set of students who propose that college

and decide whom to retain which students retain to the next step and reject those others.

the students then, move in the next step, those who are rejected, would basically repropose to other colleges and move on like that.

which college then decides so where there is no firing constraints manifest itself

it manifest in different way there

so if the college has to decide to reject of its given students in the beginning round, he would never have any regret about the decision later on.

because, remember unlike, I mean, in the college proposing deffered acceptance algorithm, set of availiable choices are huge on the proposing side.

and minimal for the receiving side, in the beginning, but their fortunes are chaned in the opposite direction.

as time goes, the choice availiable for the proposing side shrinks

and the choice is availiable for the receiving side expands.

so in this student proposing deffered acceptance algorithm, essentially students have a lot of choices

and their choices shrink, the college start with a very small number of choices, but their choices increse.

so those students who the college have decided not to like,

I mean, decide not to accept, those student that the college didn't like when the choice is small,

they don't like it when the choice is expanded.

that's the countra positive of the substitatubility condition

so go back,

in other words, we could have stated same condition, on the countra positive form, meaning that not this implies not that.

so if you don't like, if the college didn't like students S, when the choices are limited, continues not like it when choice expands.

so again, even in the student proposing deffered acceptance algorithm, the substitatubility means that there manifest itself as a no regret feature.

Another was that, this college never, at any point in the time, in the algorithm, regrets decision reject student early on. Okay?

In the college proposing case, it never regrets to propose to somebody. Okay?

So that's the difference.

[45:00]

Now, just to give you better intuition, here is an example of nonsubstitutable preferences.

Have you seen that?

So the hint is, see what, that whose preferences not substitutable.

How could you make the substitutable if you are able to, you are allowed to modify the preferences in the minimal possible way?

How could you make it substitutable again?

Yes, exactly. That's the thing.

Because here, with the three available of this S1 available, he likes the 3, with S1 unavailable, he actually doesn't like S3. Okay?

So this element to be complementarity.

So here, there exists no stable matching which is not simple.

Actually we have to do somewhat to show this.

But at least you see how you can run into trouble.

To work, to run one of these different exercise of algorithm. Okay?

(Student Questioning)

No, there is no problem.

In fact, we are going to show in Hatfield Milgrum that you are going to use general motion of core.

In other words, you are going to allow for …

So there are two issues.

One having to do with the fact that …

So you could be that are multiple coalition, let's say there is coalition that is blocking the current matching.

The coalition includes multiple colleges and multiple students. Okay?

In such a case, there has to be one college and number of sets that should also block. Okay?

And that has to do with this underlying structure which is the two-sided matching structure. Okay?

Without the being of a college and the associated students that the college is matched with in forming a coalition, that cannot be blocking coalition.

So we can break down to the level of single college and multiple students. Okay?

That part is without loss. Okay?

The second part which is kind of less trivial is that, the coalition may not include all the students that the college is going to form, is forming a coalition with. Right?

So only the new students that the college is allowed to retain some of the students from their old matching, and add students and form a blocking coalition. Okay?

You can expand the notion of coalition to include those who are retained.

New coalition that's dominating the old coalition may include those agents who remain indifferent.

Not every member of coalition will be strictly better off.

The same discussion that we had before. Okay?

So it was called group stable. Right?

So any matching that is robust against such blocking coalition is, we called it group stable matching.

And also we called it strong core. Okay?

Strong core in the sense that we allow the coalition, blocking coalition to include those who may not be strictly better off when we define the notion of blocking. Okay?

So you don't lose anything by just focusing on a pairwise in this case. Okay?

What I am saying is that the matching is going to be also in the core, strong core.

Okay. So here is an example, so let's imagine that we run a deffered acceptance algorithm.

Okay. So let me probably useful to write down, copy down.

Actually, let's do the … let's run the algorithm and see what happens. Okay?

So it's college proposing, so the fixtures are students.

[50:00]

So we write down the students S1, S2 and S3.

And the first step, the colleges make proposal to the most preferred set of students.

For college 1, it's S1 and S3, right? Okay?

For college 2, it's also S1 and S3. Okay?

Now let's see what happens with S1 and S2 ... S1 and S3, actually, right?

S3 likes C1, and S1 likes C2. Okay?

Okay. Now both of them potentially can make a decision. Okay?

So college 1 sees S3 rejecting its offer, no.

S1 rejecting its offer. Okay?

With S1 gone, what's the choice?

S2 is the choice, right?

What about college 2?

S3 rejected its offer. Okay?

S3 is gone, so therefore the next bus is S2, right?

Now it's S2's decision. S2 likes C2 better than C1.

Before doing that, one is here.

C1 decided to propose to S2. Okay?

It is okay to retain S3. There is no regret to have S … Because S1 is gone here already.

Without S1, it was okay actually, the third best in fact ... The best remaining

So the requirement not to be able to retract its offer to S3 was never binding at that point. Okay?

But in the second stage, notice that S2 rejects C1. Okay? And accept the C2.

So there is nothing for C2 to do, because there is no rejection.

C1 now, what does he do, C1 seems that now run out of all the students that he can propose to.

But notice here that it's now regretting. Okay?

S3 is never acceptable. Okay?

So it's not even individually rational.