Title: 시장설계이론1, 다대일 매칭 (1)

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- ✓ **Dictated: 강은경, 강성호, 김신희, 김종백, 신원대, 현소형**

[00:00]

we are gonna game??

so for those who came a little late, we will move to ten'o clock start.

starting this wendsday from ten to twelve.

so um..

what I'm telling now to, is the many to unmatching setting.

so trying to sort of relax the assumption that the what gent, each agent is matched with, is matched with only, can only be matched with a one agent on the other side, okey?

so I guess that the mutual setting to think about this, think about many to man, one matching is college admission's problem.

again, I mean, we're gonna saying that you're not using this as the model for describing the college admission's process.

because many cases use this are done, decentralized fashion.

that is not, that is no real centralized machanism that produces stable matching, okey?

in the real world, except for some contries like in china where precesses centralize.

so there you might this is more applicable.

it's just a name, at this point.

so that is two sides.

only one side, we are not calling ,you know, man and women now.

one side, we have colleges.

on the other side, we have, I said workers but what I should said is students, okey?

because, okey, so because I mean often college's sides is only called firms, okey?

student's sides often called workers, okey?

that makes any difference.

so in the hospital matching, madical matching setting, colleges are hospitals and a workers, you know, students are doctors, okey?

so, so far the same as before, the matching is same as before.

except that now we have an additional information or primitive or structures to describe the underlying environment that is the that you know, since each college can be matched with more than one student, okey?

that is description of how many students that each college can accommodate.

and in other words that the college comes with capacities.

so QI is the maximum number of students that college I or CI can accommodate, okey?

and then matching is then described by this sort of function just as before.

mapping from the set of all agent.

so sort of way to think about it is that one way to think about is that each college has like a multiple clones as many as, as many as its seats, okey? Which is a size of capacity, okey?

we can think of this as a copies, you know, copies of the colleges or think of as a clones, okey.

it will be use.. come out.. turns out quit useful way to think about it as you see later.

so matching is mapping from set of all agence into a set of all agence, okey?

such that whenever..

a student is matched with collegeC.

okey?

so college C admit sutdent S, okey?

and that student S that student S must be matched with college C, okey?

so this is just some sort of consistency requirement that we, we had before in the man and women.

in the marriage problem, I mean, the only difference is that now we have sort of set is an element, okey?

symbol here.

because college can admit more than one agent, okey?

and

so the other thing is that all C and S.

okey.

college must be.. Okey.. So..

okey,so students are matched with college C, I mean, must come from students, of course.

and then each college can not admit more than its capacity, okey?

and wherever a student is matched with college C.

any element of agence that college C is matched could be either, a student or itself, okey.

so when , when we say C is an element of mu of C.

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we simply means that seat that particular seat is remain unoccupied, okey.

so unmatched.

so we all rest is idol an element of c idol student is idol match with the college or match to himself or herself

I guess that this is not quite right I mean this is I guess that this is the S union C right? S union of small C ok?

that's a very good point

so I think that he is asking me about why we are insisting equality

ok because he seems that some seats might be remained might be unoccupied on a sign

in fact that possibility is allowed

because when we count C so for instance it could be that

let's say that 3 seats ok ? Student 1, student 2, and c ok?

so you always make sure that this equality holes and still accomidate possibility that some seats might be unoccupied

because by simply filling the unoccupied c seat by c ifself ok?

so that could be multiple cs

so it could be like that uh c is four and I mean this algorithm change right?

this is called multi set in fact ok?

ok now so the next business is two discribe preferences for on the student side describing stduent preferences just this before

we released we focused on ordinal preferences and so we released the ordinal preferences which student just this before

for college side however it's a little tricky

because what college demand is not individual is a set of individuals ok?

so to redially accurate to redially general we have to set of a think about the domain of college preferences to be entire powere set off students

in other words entire set of subsets of students ok?

so you could for instance imagine college having ordinal preferences of a different subset of students ok? Like this

as it turned out that we cannot accomidate the fully general preferences of this type even the ordinal preferences

so you need to gurantee the existence of stable matching

we therefore have to make some restrictive assumptions about the kind of preferences we will allow

we will set off a stock very easy case we will make a very strong assumption in the beginning

and we will try to relax the assumption a little bit ok?

the assumption that we will make essentially allows us to describe college preferences just as student preferences in terms of individual's ranking

so under one circumstance is college preferences represented by simply a list of individuals

well this is a kind of assumpion we call what their goal response preferences simply means that for any two any pair of students

sns prime ok? There is a clear core ranking on the part of give the students college

which is unaffected by who else ok?

idol than the students the college admit ok?

for instance suppose that there are any students sns prime ok?

and subset as tilde ok?

such that this um as tilde said each cardinality is the rest of capacity

so you can hire you can admit one more student at least ok? After S tilde

now college this college c likes s we'd like to admit s in addition to s tilde ok?

more than it does so we just respect the s prime if and only if college likes sns   prime

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so there is an individual rise ranking between the two students so such that no matter what other students that college admits

look at the ranking is the preferences between the two students over this two stduents ah is not affected ok?

in other words that college preference can be completely described by a fixed ranking over individual ranking of students ok?

so in other words that college would like to admit in their order of the rankings ok?

that's a restrictive assumption

we show exactly what kind of the situation cannot be I mean this is maybe may sound reasonable but may not be some other cases ok?

it is not fully general ok?

and also each student whether or not the student will like to be added um the college would like to add the student

that's not dependant upon who else the college is admitting whether this individual rationality also is dictated completely by this individual rankings the rankings of individual students

given that in fact there is a very simple trick that you can use um that before that

I mean let's just go over the main stability concept

so we say a matching is individaully rational if no agent match is unacceptable to the other side

a matching mu and so that's the standard individual nationality

we can talk about student being unacceptable without again without worrying without having to worry about what other students set of a student this college might admit ok?

as long as the set it's cardinality of the set is the nest capacity of course

because of this responsiveness uh assumption

given that a matching individual rationality concept is precisely the same as before matching mu is blocked by a pair c and s if they each prefer each other two they are matches on the mu

in other words the c likes this student for s prime morethan s prime which is in this set ok?

so given match mu assigns number of students ok? To scholl c to college c ok?

and number of students could be empty seats as well so

here s prime could be c simply ok?

in case the mu c does not invite some empty seats and vacant seats

in any case so if c and s are a pair of college and a student s is a blocking pair

if that college would like to replace one of her, one of its assignment, its could be student or an empty seat ok?

with that student ok? Which is well defined based on this responsiveness preference of the assumption

and at the same time the student itself we'd like to match with that college more than his current partner current match right?

the same notion accept that you need to recap for here because s prime could mean an empty seat ok?

so currently there are some vacant seats left and then l like this student acceptable in each case that student can be part of the net of blocking pair

so a matching mu is said to be a pair wise stable if it is individually rational and is not blocked by any pair of agents ok?

any pair of a school, college and students ok?

one could think about just as before sort of stronger concept I just called group stable or strong core

and we tell you why we call strong core a little bit later

it's basically means that they should not be any blocking collision or bigger sides

if there is no subset A of agent and alternative matching mu prime ok? Such that its agent, its college in the collision ok?

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should be matched one the mi prime to somebody who is idol in other members of collision or somebody who is given by the original matching ok?

and each agent must be strictly better off on the mu prime than mu ok?

so you can imagine collision consisting of let's say a college and number of students ok?

all we require is that everybody in the collision to better off on the some alternative matching they can come up with themselves

given the possibility also that college that is in the collision is allowed to retain some of the students from the original matching ok?

if I am given let's say in the original matching this must like my assignment I get to keep some of them if I like ok?

but I could add new student ok?

so that's a stronger notion because in two senses

one is that we are allowing the collision to be bigger insides than just a pair which is what we are considering here a pair of stability that's one sense

another sense in which this is stronger is that exactly that if you sort of think about sort of collision true collision here is actually A plus sub agents who are part of the matching with the collision

another is that let's say that this was original matching and Cs forming a new collision the blocks the mu by having S3

let's say S1 and S2

in fact so and then C this is the new matching C right?

so the collision here is what uh C and S3 ok?

so C should suppose that C strictly better off on the mu prime ok?

S3 is strictly better off on the mu prime

what about S1 and S2? Ok?

they are not officially part of the collision but they are kind of involved indirectly at least in this collision

now they are retained by C so comparing with mu this matching mu they are indifferent ok?

so the right way, another way to think about it is that collision is in fact bigger one ok?

this is the collision this is the true collision except that not every member of the collision is tricky better off

and in other words everybody is weekly better off but somebody is strictly better off ok?

that's the kind of collision

so whenever mu prime so whenever there is a blocking collision in this sense we say that collision dominates the original matching while location we are weak domination

weak domination means dominate in this sense

some members are maybe strictly better off but that could be members who are only weekly better off while indiferent

so clearly that you know allowing more collision to be a set to be blocking ok?

we are strengthen in the notion

I mean so weak domination via weak so blocking via weak domination makes the resulting concept stronger sensus ok?

we are allowing for great possibility of blocking ok?

therefore robusted requiring the matching to be robust against ok?

great possibility of blocking means that the result concept is stronger ok?

so in this sense that this mu is strong in two sensus

one we are allowing the collision to be bigger insides second allows for this sort of weak domination kind of collision ok?

it turns out however under the assumption of responsive preferences responsive preferences

these concepts there are there is no distinction they are the same

so they are not going to lose anything by simply requiring the matching to be a matching to be a pair of wise stable

if a matching is pair of wise stable then is also group stable ok?

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so the first sense that it is not actually it's not going to make a it doesn’t make a difference is not also new in the sense

not surprising because whenever there is a blocking collision that is bigger ok?

there must be always blocking pair in this sense ok?

given responsive preferences ok?

because whenever there is a blocking collision there must be a student that this college involve in the blocking in the collision must tricky prefer ok?

so if college is in a blocking collision in this sense ok? Then then must be a student in the collision that the college strictly prefers to have instead of somebody his already having it is already having ok?

so therefore you can come up with the pairs essentially ok? There is also blocking

and dwells for only for on the responsive preferences because whether I like somebody or not cannot be unambiguosly defined without responsive preferences

so I am going to just make a remark here

now so far I mean it seems that many to one matching seems much more daunting, much more difficult

but in fact there is a simple trick that one can use to reduce the many to one matching to an one to one matching problem.

so in some sense, we are going to recycle all the rusult that we have learnt after going through this way of converting many to one matching problem in to an one to one matching problem.

So, how do you do that?

the trick basically is to think of each copy or each seat in the colloege as an independent agent.

To do so, of course, it's easy physically, think about it.

college one has 10 seats, there is a one agent, which is that college.

think of there being 10 different agent.

once you set of think of, interpret this many to many matching, in this sense, into one to one matching.

so far is good, but, the only sort of issue here is the what are the preferences.

we have to define preferences.

so first of all, it is easy to define the preferences for each seat.

okay, because each seat is part of same college.

so, most have the same preference as the on the line college.

given that responsive preference , there are ranking preference , rankings of the students.

then you can, sort of, impute to, as the preferences of each seat over the college.

that part is easy.

what about the student's side?

student has a clear ranking of colleges.

then what about seats?

there are 10 different seats on the college 1.

what's the preference within that seat?

isn't the sense that student must be indifferent.

because you don’t care which seat you get as long as you get you in same college.

but in some sense, we have to reliance on some arbitrary tie breaking to come up with..

because, it means that in other to apply the frame that we had developed, we need for the most part, strict preferences.

therefore ,you have to come up with some sort of a preference ranking for every agent.

on the other side now, we decided to interpret to each seat ans an agent

you must have some definition on the part of students about the ordinal preference of different seats.

so how do you do that, you simply put serial numbers

if there are 10 seats, then you number then from one to ten,

and then every student prefer to be matched with the lower index seat than high index seat.

so that will do the job sincerely.

here is that endow each position with a same preference so , calling position here

and then, now define student's preference for alternative position

we just label each seat by numbers, once through as many as the number of seats here

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and then we , sort of artificially create preference on each students that prefer the lower index seat than high index seat.

then, we have a well defined marriege problem

so, see Cbar here, so you have like, if there are as many agents as the number of colleges in the original many to one problem]

we have many many agents

so Cbar is actually unpacking of all the colleges, you know at the level of seats

you may say, of course, Pbar is here proflie of the preferences defined in this way.

if you give me an original college admission's problem, now there is unique way of defining what we called a related marriage problem.

and then I use the result we can get, a matching of the collage admission problem is stable if the corresponding matchings of the related marriage markets are stable.

so they say collage admission problem, C.S.P. and then there is a corresponding related marriage problem induce by that.

now suppose the found the matching, mubar, and then that is stable.

in this one to one matching problem.

now, quite clear that all the stability required, we have here can be satisfied.

so, for instance that , so think about any possible, reinterpret matching here.

so once you here mubar, you can go back to mu to corresponce to that and same that it is stable.

in fact that, the requirement of stability is stronger here than here,

because, we are even talking here about there being no blocking pair with the two different seats of the same to different positions of the same school

of course, different schools clearly, the stability here requirement here is sufficient for the stability there

if it is stable, there's no for any else, there is unmatched here, so it could be labeled as C.I.,

we know that there is unmatched pair of one collage position, and student, we know that this is true, either, for any

let me say that for any, or right?, this is true

of course, that this means that , or, okay?

so therefore, clearly for any, another words, this condition is stronger than that condition

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because actually, it includes stability requirement for the original collage mission problem.

one side is quite easy, the other side, suppose you have the matching, that is stable on the collage admission problem.

now again, different collage stabiliy here is also maintained.

that's not a problem.

what about the within position?

here, we can come up with actually, let's say the collage here, C is unmatched .

a particular collage is matched with number of students okay?

let's say one through Sm

suppose that this is the number of this set of students, collage C is matched with on the matching mu which stable on the original problem.

now, we simply called the positions these guys are matched with okay?

we simply label them, in the right way

so what is right way?

now you can think of really, the ordinal preferences of collage C, all those students that collage C is matched with

line them up in the order of its preferences.

suppose this happeds to be the order.

we just simply called the position occupied by this guy to be the number 1 position, this guy to be the number n position.

then, if the label of the positions of that collage in this way, then you will be guaranteed to have, here, stabaility as well, within as well

because there is no way to block this sort of matching, within given collage

so this is a very nice result.

and all the important result have come out of this, this observation.

in fact, there is one that is redundent, so we just a strike one of three quaters instead of four quaters

so first of all, we know that there is a stable matching.

because you already know, that every marriage problem has a stable matching.

start out with this collage admission's problem, as the problem you care about,

now you converted into one to one matching problem focus that on the related marriage problem

we know that there is a stable matching okay?

found the stable matching, we interpret it, you go back, mu, and that is stable on the this matching

so there is stable matching, you can find it using student proposing deffered acceptance, or collage proposing deffered acceptance.

when you do collage, however, you should, sort of, treat each position as seperated entities.

so far, when the existance part is clear, now this part is somewhat may be not as trivial, but it is also true.

student proposing DA produce student optimal stable matching.

So that part is quite clear because, again, whatever is best for students here, must be matched here, in the reason the set of student matching

in other words that the entire set of stable matching that you have on the this original problem, exist here as well.

and we have a clear result based on what we had earlier, the result we had earlier

there is a matching, which that is student optimal, within the set of stable matching in this related marriage problem.

based on the result we had from before, and you can find that matching, by learning the student proposing deffered acceptance algorithm.

and likewise, we can learn collage proposing deffered acceptance algorithm, and find the collage optimal stable matching as well.

but here again, you have to treat each position as seperated entities

that's not a problem.

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and finally we also know, one-sided strategy proofness must hold as well, at least for the student side.

because, we know that essentially this is exact same problem.

Therefore, for the student side, the student proposing deffered acceptance algorithm gives doniment strategy incentive for students, each student to rank truthfully of different collage.

so that part is so far so good.

it doesn't carry of all, the part that doesn't carry of all, is the strategy proofness of the collage optimal stable matching.

if you learn the collage proposing stable matching, collage proposing deffered acceptance algorithm, unfortunately, you are going to get strategy proofness on the collage side.

here is an example that illustrate that.

you may wonder how you learn collage proposing deffered acceptance algorithm.

now you can do this, of course, we can treat each position seperately but more intuitively you do the following you have each collage proposing to students but up to each capasity.

if the collage has 10 seats, then at each time the collage propose number 1 through number 10 student, but no more

and then, you should make a decision about whether or not to accept or reject that proposal

those who accept the proposal, let's say, the collage has 10 proposals made, 5 return as rejected

then the collage allow them to make 5 more proposals to different students.

and then whenever there is a rejection you replace that you sort of propose as many as those rejected

I did that everything is exact same

what about student proposing deffered acceptance algorithm?

it works as follow, this is very similar to what we had before

but the only difference here, you think that you need a liitle bit carefull about, it's collage side.

whenever it makes acceptance decision, it can only even tentatively, accept only up to the capacity.

if there are like seat is less than 5 and there are 7 proposals then, the collage must accept at most 5, okay?

and then must reject 2.

if there are 7 acceptable proposals, then you accept in the ordinal preference, according to ordinal preferences of that collage right?

up to 5, and then reject 2.

and each round, that's what it happened

let's one collage proposing deffered acceptance algorithm as see what happens here

so writh down collages here, 1,2 ,3   and students are first , actually no,

in this case, collage is proposing so let me write down the receiving side

there are 4 students, s1, s2, s3, s4

and the capacity of the collage one is, I should have say here, it is actually 2.

the other collage can admit only one student let's say

collage one first of all, proposed to s1 and s2.

so, collage one here, collage one here.

college 2 is proposed to S1 here as well. Ok?

college 3, only one because the call out for college 2 and 3 is equal to 1. Call out for college 1 is 2. Ok? That's it. Right?

Now, the only student who has received multiple proposures is S1. Ok?

S1 prefers C3 of C1. And C1 of C2. C2 striked out. Ok?

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So, college 2 then turns around and now proposed to the second step proposed to S2. Ok?

Now, S2 has two proposures. One half from the first step. The second, the new proposure from college 2.

So, S2 likes S2 of C2 of C1. So C1 is striked out.

Now, college1 at this point, realized that only one of proposal is accepted. So, he has one remaining seat. He can make another proposure therefore.

So, C1 is S1 now, and then S2 is gone. So, S3. Ok?

Now, S3 now C is that he has multiple proposals. Ok? Now, C1 and C3. Ok? Like C3, C1 better. Ok?

Now, C3 has rejection. So, has to make a new proposure. Ok? Now, S3 is gone S1. Ok? That's next to student.

Now, S1 now has two proposures. C3 of C1. Ok? So, C1 is rejected. C1 has one more proposure to make. Now, S2 and S1 are gone. So, and then S4 is remaining.

Ok? And here is the matching Ok? So, that's maybe algorithm stops here. So, C1 is matched with S3 and S4. Ok?

And C2 is matched with S2. And C3 is matched with S1. Ok? In fact, this is unique stable matching which you can confirm again by running the other side. Ok?

running the student proposing deffered acceptance algorithm. Ok? Let's do it, Ok?

So, students are all propose. So, S1 proposes to C3. S2 proposes to C2. Ok? And then S3 and S4 propose to C1. Ok? And that's it.

Because C1 can accomodate 2. Ok? It will accept as many as its seats as long as they are acceptable which they are. Ok?

So, it gives you a produces same matching and that's the unique stable matching for the based on the argument that we made before.

But, suppose that C1 lies by reporting S1 and S4 to be the only acceptable. So, getting rid of these two. Ok? What happens. Ok?

Under college proposing deffered acceptance algorithm in this case. So, we have to do it again. C1 proposes to S1 and S4. And C2 proposes to S1.

And, C3 proposes to S3. Ok? Now, so, S1 has received two proposals 1 and 2. So, rejects 2. Right? Because two is worse than one. Right?

And therefore C2 must make the next proposure which is S2. Ok? At this point, the algorithm stops. Ok? Now, C1 gets S1 and S4. Ok?

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So, college 1 strictly prefer this matching than this matching. Why? Because S4 remains same but, S3 is replaced by higher ranked student. Ok?

So, it turns out therefore with that stragegic proofness for the college side is not guaranteed by running a machanism that always selects the college of two math table matching. Ok?

In fact, what is also not true is the weak pareto potimality is also not guaranteed, although we can not see it here. Ok? Because in this case these guys are also the same.

But, I can give you an example which in fact should have work to make both of these points. This is like the matching fairness kinds of example.

Students 1 and 2. I'd like to say C1, C2, C2, C1. Ok? And colleges. Two colleges. College 1 has true quota equal to 2 true quota can capacity, true capacity of the college 1 is 2.

But, likes students in this way has to choose only one like S2 more than S1. College 2 likes S1 more than S2. Ok? So that matching fairness.

Except for this, this is exactly the example that we used to establish that there is no streched proof stable matching mechanism. Ok?

So, now, suppose that's college proposing deffered the acceptance algorithm with this. Ok? When everybody is telling the truth. Ok? So, in this case, first of all, we have to wright down this way.

So, college 1 proposes to both of them. And college 2 proposes to S1. Ok? Student 1 picks the better of the 2 which is C1. Ok?

Right? And therefore C2 has to make another proposure. Now, S2 has two proposures, C1 and C2. S2 likes C2 better.

Ok? So, this is the matching. Ok? So, this is the matching that has same indexes here. So, each student is matched to his or her best school.

Colleges, however are matched with worst. Ok? Now, Suppose instead of telling the truth, now we are running the college proposing differ the acceptance algorithm, but now suppose that college 1 lies by saying that my capacity is only one instead of 2. Ok?

Basically that is equvalent to saying this. Getting rid of the first. Ok? I don't know admin more than what. Ok?

Now, without this, this is exactly what? This is exactly same problem. The men, women and matching fairness types situation before or be new that if you run men proposing, you get exactly these. Man gets the first, best.

And these are women essentially. And but if you run the woman proposing which is like college proposing, then what do you get? They get the best. Ok?

So, by lying, by saying that capacity for college 1 is equal to 1. Ok? This is the sort of manipulation Ok? Which means that by getting rid of these.

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The matching that induced is S1, C2, S2, C1 which is this matching. Ok? And so, therefore both men are better off. And both colleges are better off.

And particular college 1 is strictly better off by doing this. Ok? So, this is another example showing that it is incentive to lie. Ok?

Even if you ran the college proposing deffered acceptance algorithm. And college has an incentive not to tell the truth. Ok?

And, second of all, this example also illustrates the failure of weak pareto optimality Ok? Why? Because let's call this mu prime. Let's call the original matching.

So, let's say this is college proposure deffered acceptance under truth first. Ok? That's what? S1, C1, S2, C2. Ok?

That's what the college proposing deffered acceptance algorithm produces. Ok? This is alternative matching. Let's call this mu.

Mu is individually rational clearly for everybody. Right? Nobody is unacceptable in this case. Ok? And if you think about the college side. Ok?

Even though we comparison, what we are comparing with here is college proposing, college of this stable matching. It is strictly dominated from the perspective which college uniformly by this.

Because both colleges are strictly better off. Ok? Under this matching than under that matching. Ok? So, shows that weak pallet of optimality for one side, for the proposing side. That's whole here in this example.

So, the all the results that we have established carry over to the many to unmatching side. Anymore than important results basically except for this 2. Ok?

Weak pallet of optimality and the one sided strategy proofness for the college side. Ok? So, in some sense, So, here is a kind of related episode if you will that medical matching in the US, maybe in Korea as well.

US's Gale Sharpley's deffered acceptance algorithm. Ok? In fact, Gale Sharpley's paper came out. Gale and Sharpley's article came out in 61 or 62. I forgot exactly.

Now, the hospital system in the US used their centralized matching that was used in before in 50s to match doctors to hospitals.

And the algorithm what they used wasn't cleared that is one of these that corresponse to one of these deffered acceptance algorithm by turns out they did it. Ok?

In fact, the mechanism, the algorithm that they were using was precisely the same as the hospital proposing deffered acceptance algorithm. Ok?

If we just look at the description of the algorithm that was used is not obvious. But, there is later paper that came out and proved then in fact that's exactly the isomorphic to college proposing deffered acceptance algorithm.

Later in 80s, R Roth was involved in consulting for the clearing house. And, he actually managed to convince them to switch from their orginal mechanism to doctor proposing deffered acceptance algorithm.

And, one of the reasons is this. Because I mean we have to sacrifice the incentives on one side in any case. Because we have this genering possiblity results.

But, at least we can maintain a good incentives on one side by using the saying doctor optimal stable matching mechanism what doctor proposing deffered acceptance algorithm

But, the orginal mechanism which was equivalent to hospital proposing deffered acceptance algorithm is not strictly prooved for either side, because of this example. Ok?

So, that's one of the reasons. I mean there is true interesting story is there is that it is quite remarkable that the engineer came out in the 50s with mechanism that later shown to equivalent to Gale and Sharpley's deffered acceptance algorithm.

[55:00]

It was even invented in before it was recognized by academics. Ok? It's like you know.

I mean, that sort of question about really whether they are the ones who should deserve the credit and still I mean they are the ones who recognize the desirable properties of this algorithm. So, in some sense, the credit should go to them.

If you read the book by Ross and Sormire, they make a point that is resist earth that you know. Even Columbus discover the America. I mean there people living there already Right?

So, it's somewhat like that for Gale and Sharpley. The mechanism was already in used. It was them who discovered desirable properties of the mechanism.

So, here is a little bit of discussion that I like to have about relaxing and this responsiveness assumption.

Because so far, we made this assumption about responsiveness which made lives easy in the sense

that all we needed was to recycle what we already knew from one to one marriage problem

that is away to convert many to a matching problem into one to one matching problem.

That only works, however, for responsive preferences. Okay?

For more general preferences, that trick doesn't work. Okay?

So, here is little bit of a generalization in this, along this line.

But I am not going to do fully general discussion here until next class.

In fact, the paper that I will be presenting, discussing next on Wednesday is the Hatfield Milgrum (2005).

So you should read this paper.

It's not easy.

I am hoping to make it more accessible through my slides, my presentation.

But you should, nonetheless, try to understand as much as you can from this paper.

The reason that this is more general than the discussion that I will have today and even remaining like 10 minutes or 15 minutes is

because not only do they relax the preferences of the hospital side, what is college side, to include what we call substitutable preferences.

That's sort of the maximum extent which we can relax.

Because if you cannot relax beyond substitutable preferences, because if your preferences are not going to be substitutable, then you may not be able to get on guarentee existence of stable matching.

You lose stability in general.

So they have that.

But on top of that, they allow there to be some contracts or so that are determined through the matching process.

So when we imagine doctors getting matched with the hospitals is not just the matching between this agents that happen.

This also the salaries and other terms of contract that get determined through this matching process. Okay?

So what they come up with this more general framework for understanding matching which not only match hospitals to doctors, but also determining the contract terms. Okay? In the process …

So lot of us sort of said that contract terms that are stable in some sense. Okay?

So just most general … so if you reestimated the state of the art paper ...

So once you have understood that paper, you have understood really in some sense the frontier of the research to a large degree. Okay?

Which is a great thing, I mean you didn't like three, four classes, you have not reached the frontier which is cannot be said about other, I think, topics very easily.

Anyway, so what I am gonna do is just to illustrate what kind of problems you may deal with, you may run into if you want to do relax preferences. Okay?

And also, you know, little bit of preliminary discussion about somewhat simple, okay somewhat intermediate step between responsive preferences case and Hatfield Milgrum case where we deal with the contracts.

[60:00]

So here we are not going to deal with the contracts. Just the standard many to one matching.

But we are gonna just relax the preferences. Okay?

So to relax preferences we have to deal with with colleges preference of a, or possibly substitute of students. Okay?

So we are gonna for students are, just as before, each students has a clear ordinal ranking of colleges

Each college, however, has ordinal ranking, define over the power set, meaning that the of all subsets of agents.

And the way to describe preferences by way of, by using this what we call choice function. Okay?

Choice function for … this is the entired sign of students.

For any subset as tilde, okay? choice function for college C, okay? picks out the optimal subset of S, optimal for the college. Okay?

So now there is that give me any subset of … so you are limited already, right?

You are limited to choose from subset of students.

Within the subset, okay? For any subset that …

So gives you the optimal subset, optimal in the sense that any other subset of as tilde is dominated from the perspective where college by deadline that you choose. Okay?

And the choice style is unique.

So what's the typical element here is a set of subsets given that are multiple subsets that are equally good for you then that are it's a correspondence basically.

It's not a function.

When we assure strict preferences, in other words, each college has strict ranking of all possible subsets of students, okay?

Then this is going to choice function, give the unique element, just of element happens to be a subset or students. Okay?

So genetic value of this function is, not an element, not a student. It's a subset of students. Okay?

That's the only thing to be a little bit careful about. Okay?

Now, a college sees preferences as said to be substitutable if for any S containing S and S prime. Okay?

If S is part of the optimal subset, college C chooses from subset S. Okay? Entire set.

Then this guy happens to be also part of the optimal choice when the college loses some other students S prime. Okay?

So if college likes to choose this student, okay? The college would like to still have the student when some of his optimal choice is … when he loses other students from the set. Okay?

If something is a part of optimal choice within a bigger set, and if you face a smaller set containing that element, you still continue to choose that. Okay?

It seems that this is like a statement of irrelavance of inferior alternatives(IIA), but it's not.

Because, let's say that this is the set and this is your optimal subset. Let's say this is set. Okay?

The student here would be probably optimal choice.

Whenever your faces are underlying set, that's [?1:04:25]   on that. Okay?

That's just reviewed preference, nothing more than that.

You don't need to, you don't need any assumption about it.

That's the situation where you need an assumption, is when you lose some part of here.

This is already grown.

What it says is that you continue to demand that guy even in that case.

So maybe to give you an example, so let me give you an example.

[65:00]

So Chc of, let's say, first presents S1, S2, S3, S4 is let's say S1 and S2.

So you need best among the set if you are allowed freely choose anybody from the set of these four people, then you choose two. Okay?

Now, what I am saying is that there is more assumption involved. Okay?

This just comes out from their beings to ranking.

Well defined true ranking of all possible subsets of students by for college C.

The statement that requires non assured assumption, substitutability assumption in other words this is about substitutability binds is the situation like this.

So let's say, something like this.

So we are not saying that … okay.

Even while you lose S2, the college C will continue to demand S1.

Substitutable in the sense that you know there was that the you intent value somebody more

When you don't have other, I mean, because everybody is from the perspective this college students are kind of substitutable.

And there was that rather possible substitute for this guy is another level you demand so much more the same guy. Okay?

That's the sense in which that this preferences are substitutable.

And what I am saying is that this is where the substitutability binds ... bites. Okay?

Because the fact that S1 is demanded when there S2 is not available. Okay? What the substitutability means …

Doesn't necessarily mean that S3 is not going to be demanded.

It is possible, S3 could be also demanded. Okay? Just because S2 is gone.

Situation is different, therefore … and of course that if this is true, then this is the solution …

This comes out really without any assumption. Okay?

This fact here requires substitutability.

So here is an example of substitutable, preferences in substitutable, in particular college mind.

Preferences of student, but not responsive. Okay?

So where does responsiveness fail?

So if the preferences were responsible, this rankings must also apply here as well. Okay?

Because when it is allowed to choose only one student, like S3 the most, followed by S2 and S1,

He prefers when S1 is admitted however the ranking is reversed, prefers S2 over S3. Okay?

So this preference is nonresponsive. Okay?

And yet these are still substitutable.

Why? Because if you are, let's say, suppose to begin on the subset including S1, S2, S3 watches up to choice you choose S1 and S2.

Both student 1 and student 2 are demanded in that case.

Now if the college here is limited to choose from only a single [?1:9:25] let's say S1. Okay?

Here is true S1. Okay?

So therefore substitutability is satisfied.

So you can check if this condition is actually whole and is satisfied, and it will hold anywhere. Okay?

The case where, let's say, so then you might wonder what's excluding here are the kind of situation.

Like this, very similar to what we had before except for one notable exception which is that S3 here is missing. Okay?

[70:00]

So when S3 is demanded in this case, when the college is allowed to choose S1, then college would like to choose S3 as well. Okay?

But then, when the college must choose only from … either choosing S3 or nobody, the college would actually choose to hire nobody. Okay?

So that's where the substitutability fails here.

This is non substitutable preferences.

It's quite clear, I mean that the definition here, the terminologies are quite suggestive where the underlying concept. Right?

Because what's the flavor here is , flavor here is that of complimentality

Because demand for student 3 hinges crucially whether or not the college can also has another student.

So I mean if the students think about this college as supporting team, like athletic team, and you should play baseball team.

So suppose that there are some pair of pitcher and catcher that work together very well. Okay?

So this catcher, he is not a good player by himself.

But he is a great player to play with this particular pitcher. Okay?

In that case, if you cannot have this pitcher, you may not want to have this catcher, either at all. Right?

So that guy select that catcher. Okay?

So demand for the catcher exist only when the team can also select the pitcher. That's sort of complementary with that catcher.

So that's without sort of complementarity of preferences.

So I want to state the main result and I am gonna stop here today.

Let me not do this actually.

Let me just say that the what we do here, the first generalization …

What we will show briefly next time is sort of redefining motion of blocking and therefore stability and the main result is going to be give a substitute of preferences, stable matching still exist.

And then we will switch to Hatfield Milgrum, so you should try to read Hatfield Milgrum

There will be another problem set and the first problem set is due this Wednesday.

Okay. That's it