Title: 시장설계이론1, 완전 정보 하의 균형 (1)

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- ✓ **Dictated: 강은경, 강성호, 김신희, 김종백, 신원대, 현소형**

[00:00]

Stable matching mechanism that always produces men optimal stable matching. so, men proposing deffered accepatance algorithm will be such a mechanism.

Would be strategic proof for men ok?

So, incentive on the part of each man , dominance strategy incentive on the part of each man to report truthly about his ordinal preferences

Of course that , you know, we don't have the same sort of incentive property

with respect to women

So, that sort of [?00:34] question that it is very important, let me just , I didn't mention this, but one application of this one sided strategic proofness the reason that this is potentially important for practical purpose is that

In many some settings, ok? ,ah, only agent on one side strategic agent

So, for instance school choice problem ok? The side of students are stategic

But the school side did not be, right?

There are kind of passive agents that act according to policy set out by the school systems

So, therefore, I mean, imagine that social ,kind of social planner ,ah,

making decisions on behalf of the interest of , persuming maximazing interest of students

So, therefore, I mean, in some settings we need to worry only about one side of , you know , in the market

which case, I mean this is a actually good result because we got all we need

Now, let in the genuine two sided setting ok?

when both sides can be potentially strategic . Then one has to worry about

this failuer of strategic proofness on both sides

And, set of raised question of what it happen in equlibrium precisely right?

So, suppose that you learn a mechnism such as men optimal matching mechanism ok?

Then, we know that , um, the men have incentive to report truthfully , what about woman?

That's the , I mean, difficult part. What this shows , I mean, and so this result

that I am presenting in the slide and the next page , slide , sort of go together

,I sort of providing oppsite directions of the argument

If short of focus on a full informantion equilibrium. So I mean , so one can take an issue that equlibirum concept of course.

full information , instead , every agent knows the preferences of

the other agents ok?

And can coordinate , would complete nology about the preferences of that agent

So, I mean, that's unrealistic in many settings , ah , still, I mean that's all you

got in some senses

Because the, more realistic setting , one wear, each agent knows his or her own preference , I mean, men side, you don't have to worry about it

Of course, if you use men optimal stable matching mechanism

Then, whether men knows other agent's preferences or not is evaluable

You have the incentive to no matter what the agent reports which men haven't incentive to report truthfully dominance strartegy ok?

So that , men side is not a problem

But women side , the more realistic solution concept , sort of more better solution concept will be[?4:00]

Each woman knows her own preferences But doesn't know anybody else's preferences including men or other women ok?

And, probably, perhaps, some belief for some belief about distribution

of preferences

And then, try to sort of strategically in her best interest

That equlibrium , there is nothing much done . I will sort of mark a market

at the end of this two slides

So, we still don't know much about what will happen

Basically, result that we know , based on example , suggesting that anything

can go , ok?

So, you can even have an equilibrium matching that is not stable

So, supported as an equlibrium.

So, but indeed, we have some sort of clear-cut result once we adopt this unrealistic full information Nash equlibrium solution concept

[05:00]

And the result is some what stricking that is that any stable matching can be supported as an equlibrium

Well, men look for truthfully which is the other women strategy

And woman reports whatever the particular stable matching suggest, I mean, matches all

As the only acceptable men ok? So, what I am trying to say is follow

So, given that I say, you can star out with general any sort of marrige problem

right?

Everyone knows everything here because this is the full information setting

rihgt?

Ah, so men side don't worry about it , women side m let's say that mu is stable matching ok?

Relative to ture preferences

So, it's well defined concept , I mean, we know it exists because of the

Gale-Shapley result ok? It exists ok?

So, the mutiple here ok? Fix anyone ok?

What I am trying to say is that there is an equlibrium where this matching arises

There is an, full information nash equlibrium which would implement

that stable matching ok?

And in particular what are the strategy is adopted , all the men report truthfully

ok?

Each woman reports ,so for each , woman reports the man ok?

She would be under that stable matching as the olny acceptable man ok?

So, that work

That's an equlibrium , first of all, so, let me make a couple of observations

So, let's call , so this is, set is same but this is the , cadidate equlibrium

strategies that I just mentioed ok?

People are involved in the following or each man. Everybody reports exact

same as this

For each woman, you know, this man is the only reported as only acceptable

man ok?

That's what p'(p prime) is ok?

What this says is that mu which we know is stable by definition.

First of all, the observing is stable relative to p' ok?

Now, why is that ? What, the reason for that is that , remember the only thing that stops some matching from being stable , I mean that either two things

either is individually rational , irrational or there is blocking fair ok?

So, mu is still idividually rational ok? Even here right?

Because everybody, every woman is , I mean, mu could involve some

woman not matched, ok, that's fine ok?

So, this matching is still idividually rational , that's for sure , for both sides

Relative to p' pretending that p' is the ture profile of preferences

Second thing to observe is that how come there is no blocking fair ok?

Remember that this is very short listed form of preferences for woman side

ok?

In particular, in, here , in trues, there may be something better that, there

would be something worse than that.

That will be the true preferences for each woman ok?

Now, something worse doesn't create problem , cannot be used to block that

gives man to this woman ok?

Ah, the fact that this stable means that there is no blocking fair that can consist some man here ok?

There could be a blocking fair here , originally but there was not , that's how

this was stable ok?

The point I want to make is that if you sort of eliminate the man ahead of

this person , we are simply create , increse the chance of something not

being blocked, I mean this not be blocked, ok?

So, simply reduce the number of possible blocking fairs that could be active

ok?

[10:00]

Ah, so, in other words, whatever can block mu under p' should have blocked under p because there are so many more blocking opportunities ok?

Now, So now, we have establish that, therefore, mu is still stable matching

under p' ok?

Now, second thing to observe that I like to being attentioned , to attention is

that ,ah, mu is the only stable matching under p' ok?

Why is that? Because if mu is , if there are some other stable matching that

is different ok?

on the p' ok? That means that , the point is that any stable matching , right?

Any woman that is matched to some man can only match to this man here

rihgt? Because this man declare as the only acceptable ok?

Now, by the roll of theorem, that means that in any stable matching ok? Any woman that is matched must stay in any other stable matching

In any stable matching, the set of matched woman must be the same ok?

Any woman matched here, which is stable matching on the relative to p' must be matched in any other stable matching.

There is no other matching different from mu ok?

Because that , without really changing set of matched woman and unmatched woman ok?

Because the only person one can be matced to be on the this kind of preferences one that is declared to be the only acceptabel man ok?

So, that's the proof for claim that mu is the only stable matching , if it the only

stable matching , then , it must be men optimal stable matching on the p'

here, so further point that is established is that um, if this kind of profile of

strategies would support mu as the stable matching

Even on the p' , even though need not be, we have picked mu, to be any

arbitrary stable matching on the original matching , original marrige problem

Now, what we show is that this sort of preferences profile preferences make mu the only and, therefore, the men optimal stable matching ok?

Now, the next thing to do is that we should show that there is no profit of

deviation for each, every agent ok?

And that is trivially true that, I mean, not so trivial but you know true from, but you know from the

previous theorem that is that which man , it is dominance strategy ,

so it is there is not profit deviation for each man to DBA from reporting trutufully

For each woman, we need some work ok ok?

And suppose that instead of doing that, woman w , and I am reporting something like this ok? Something different

And as a result, I got, let's say, mu' (mu prime) and so p' along with p not w

produces as the the MOSM(스크린위쪽에 적혀있음) ok?

OK, I am considering the possible deviation by woman , man , there is no

incentive to deviate , take any woman and suppose that that woman w

deviates to Pw double prime and induces a new matching ok?

Which is of course MOSM relative to p' , which is what I am defining here

to be the profile ok?

[15:00]

And furthermore, of course, this woman likes her mate under here, then under here

This woman likes this man here ok? Ah, wait, and prime ok?

Now, we can immediately establish the following at this point

So, then, this man must like his mate on the original matching ok?

Given his preferences which have not been changed here ok?

Between the two matching because when we form p''(p double prime) we only change preferences of this woman ok?

In particular, all the other man's preferences remain fixed ok?

Then, two , this woman ok?

ok, why because , otherwise, this woman likes m'(man prime) over her match

on the mu ok?

And if it is reversed , that simply means is that m' also likes this woman over

his mate on the mu, which means that m' and w will form a blocking fair against mu ok?

And therefore, why is that a problem , well, in that case, mu would not be stable , um, mu would not be stable under here ok? Not be stable under there

In fact, on the here.

two, because we are still over there, okey?

And this is the true preferences.

right.

So I understand that when you write this sort of way of writing down things.

It's confusing, I mean you have to really careful about when you write down preference in

equality.

I mean, what preferences it is.

I mean we are talking about.

so in this case, we're talking about relative to true preferences, okey?

so so that this is needed, okey?

now, let's call thus women, W^2, okey?

It's man's match under mu okey?

okey?

Now, remember that this man, this woemn declare thus man as the only acceptable under this preference, okey?

And therefore when she, thus sort of women inflation.

Women, W, thus uh kind of women inflation.

And man is herself to be matched with M^2.

she took, took away M^2, okey?

And what does that mean that means that W^2 must be unmatched under P^2.

okey?

okey.

why because W^2 declare M^2 to be the only acceptable under P^2.

And then some how this other women, man is to take him away, okey.

that means that uh.. Since mu^2 itself is the stable matching, okey?

meaning individual international it has to be that M^2 must be self- matched, okey?

now, once we kno thus, then thus us second observation tha tone and two to combine to imply that M^2 and W^2 blocks mu^2.

right.

so why meaning that block mu^2 under, under Pw^2, okey?

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why because remember mu^2 preferences are always truthful, okey.

So when wqe say this is, I mean true under relative to stressful trust under P^2, Pw^2.

women, women prime right, right, declare himself to be thus guy to be better, right?

then remaining unmatched.

So therefore W^2 also to be matched to M^2 related to staying unmatched, okey?, under Pw^2.

on the Pw^2 which is exactly same as P single prime for that women, women prime.

because he, we are talking about unilateral deviation by a single women.

In other words, only other women including W^2 has not changed her strategy which is to declare uh.. M^2 as the only acceptable match, okey?

It's a little bit, a little bit complicated.

but um.. The point basically is that any stable matching can be spported as full information as you could be unmatched.

the next is kind of converse.

Now, what about.. Okey.

let me think.

okey.

what about the converse?

it is the converse.

Any equilibrium in dominate, undominated strategies arising from man proposing differed acceptance algorithm has a stable matching, okey?

so we show first, if give me any stable matching, I can come up with the profile of strategies.

that is supposed as the only equilibrium as supporting as full information equilibrium match.

now, any match that can arise any equilibrium that is undominated meaning that is un equilibrium that every man report truthfully, okey?

has to be stable as well, okey?

so if you want this game have agence play man proposing differed acceptance algorithm game.

this is a game.

it's sort of form, create a game for agence to play.

and as long as man play their undominated strategies, the dominated strategies.

the only thing that come out in the way of matching, it has to be stable, okey?

so

I mean, argument, it's that uh.. Support not.

let me if matching that arises unstable, okey?

but, so meaning that, when he say, has to be stable relative to, I mean, relative to true preferences, okey? relative to true preferences.

so what we are trying to contradict is the possibility.

that is an equilibrium profile of strategies

that is support the matching.

that is unstable relative to true prefernces, okey?

of course, uh.. I mean, since we are using this is the mechanism, okey?

any matching that will come out would be stable, I mean optimal stable, in fact relative to the prifile of strategies that I reported, okey?

so yeah, so unstable yeah, yeah.

then since this is unstable relative to P.

there must be a pair M and W, okey?

that will block, that will block mu, okey?

under P okey?

so what is that, what about possibility of, what about the possibility of stable?

what about the possibility of blocking by a single agent, individual relationality?

okey, so, in the man side, I mean, of course, that's not possible.

so, in other words that, we can not, so we can, the fact that this is unstable P can not mean that this new matching mu which is unstable, it can not be.

because it is individually relational under P

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because for man side, each man reports truthfully, okey?

for women side, even under P^2, okey?

for the women side, undominatedness means that you know, they are not report anything that is

unacceptable as acceptable even under P^2, okey?

so therefore MPDA always selects the stable relative to P^2 which selects stable relative to P, okey?

because, because everybody is playing under undominated strategies.

Now, so therefore the only possibility is that M,W, they are must be a pair that blocks mu under P, okey?

Now, so given that the women in that blocking pair can deviate by listing only M as the acceptable man, okey?

in each case, of course, she will reach um.. Right.

she will reach.. Okey.

she will reach this man that she list as the only acceptance man before she reaches her match, okey?

under mu, okey?

so therefore, and then that man will accept her, okey?

because she is, she is the um..

so this man would like because of the fact that this blocks.

man always tell the truth, remember, okey?

so this man also list her a head of his match.

so, therefore this women will be accepted by this man, okey?

that means that, that is a profitable deviation.

so give me any unstable matching relative to true preferences.

I can come up with deviation.

an agent who have a profitable deviation that can block even mu, okey?

so, so game, I mean this is kind of, result is kind of nice, okey?

that set of equilibriym if you will.

in the set of equilibrium in dominated, in undominated strategies under MPDA.

but for that matter on the WPDA coincides with the set of stable matchings under full information nash equilibrium, okey?

so if use this game, the only equilibirum gives us stable matching.

it's kind of nice result.

but relies heavily on complete information on both sides, okey?

if we relax this assumption like I said earlier anything goes, okey?

we have an example showing that even unstable matching can that is, I mean, strategy coordination mistakes, of course, in that case.

so we will not habe a stable matching always guarantee to come out as the equilibrium outcome.

so at this point, I'd like to, I'd like to move toward the sort of second part of the first face of the lectures.

namley, I'm gonna move to turn to many to unmatching.

Even though this is a liitle bit early, I think that this is the sort of logically right moment to take a break.

because this is sort of, we are changing the topic

so I'm gonna take three minutes break.

and then come back, okey?