

Title: 시장설계이론1,

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[00:00]

ok so someone this arguments you may notice they mean it's actually not easy some are intricate, subtle because the judicious and wise user, clever user of logic but the nice thing but let me make a few remarks man is that so you have to however appreciate and understand the basic frame or basic foundation you got it and thoroughly in order to be able to do research right?

i mean it's one thing just to understand what type of results you get at a superficial level reach maybe just enough to do understand what the literature says you know one of the results you can get

but you have to do more than that to be able to do research ok?

and a second thing however is that um some basic set of results that you have to understand very well that are but this nice thing that are not whole out of those ok? in this matching literature

you don't need to actually know have a real you know background in very advanced math that is no one else none of this you know fancy math used

only need is just to understand the logic carefully ok? that's all you need mathematical background necessary to do research is actually not very high it's obsessed

and last um you know in fact the fact that some of these things are maybe difficult, tedious do you understand?

cause i actually should be a good thing because you know everything is easy then you know everybody will do this right?

i found actually matching suited to be an A where it's very easy to acquire comparative advantage in terms of doing research because actually you don't need a whole lot to be able to do research as long as you work hard to understand basic frame uh very well

um and then somehow some of these tedium and difficulties are enough to deter out of people from doing that means that you still have and can do some research and do well ok?

so and we go back to it's stop preaching and go back to the teaching

so so far discussion that we have had had not included strategic issues ok?

so there are no questions about whether or not agency would report truthfully ok?

and for those who are familiar with the standard mechanism design literature I'm assuming that you guys are very familiar because of you know professor kim probably having touched you well in the mechanism design.

The big part of issues, you know , big issues there are all related

strategic issues

Incentive comparability and strategic issues and so on

So we haven't discussed anything at all about the those OK?

So, what can you say about those, well, um

It turns out that we are not going to, you should, at this point,

expect that ,you know, we should not be optimistic about getting

very positive result in terms of incentives. OK?

The desirable matching, in this case, desirable allocation, ah, you

should not expect that to be achieved, you know, way that is sort of

dominance strategy incentive comparable frames that

In a way that provides good incentives for agent to report choose free . Ok?

Because as long as, the implementation ,solution concept,

or the notion of implementation is that a dominance strategic implementability.

We know that, in general setting , there is seven to eight impossibility result

suggesting that ,you know, ah, you can only hope to get dictatorial outcome implementable.

In general, without free any restriction on the preferences , underline a structure

But our situation is not quite bad general ,we put some structures. First of all,

we are focusing on two sided matching .

And second of all, we often impose restriction on the preferences in terms of

strict ordinary preferences and so on.

But the matching set of , put a lot of restriction, so, that is some hope,

in general, you can do very well.

You can do probably, not as well, not as badly as [04:53] theorem we suggest

Because there are, in fact, you know, we are not putting in [4:57]

any restriction of preferences

But at the same time, here is an environment well, we can't necessarily

use the monetary transfers

[05:00]

If you were to allow, ever to use monetary transfers and preferences

of agents, ah, those are, we let say quasi linear preferences.

We know also that ah, Vickrey-Clarke - Groves mechanism, VCG.

We will get you implement, we will get you the efficient

allocation implemented in dominance strategy. Ok?

Here there is no mention of any monetary transfers.

So, from, in that sense, our anticipation at this stage should be perhaps

, not everything nice can be implemented in dominance strategy. And that's what we will see

That, here are underline, sort of rotation, you know, terminology.

we already define what mechanism is. Mechanism is a mapping

from the sort of arbitrary marriage problems into the set of matchings. OK?

Give me any marriage problem.

I will give you A matching. Ok?

That's what we call mechanism. Anything that does that is known as,

it's called matching

Here is a mechanism

Here is, I am just listing, to rotation, to preferences. ok?

I am fixing already set of men and set of women, just varying

the preference. Ok?

Just start with the environment way, set of men and set of women are fixed ok?

Now, mechanism here is simply mapping from possibility from profile of preferences. Can you imagine?

Into a matching. Ok?

It doesn't, the distinction doesn't matter ok?

In fact, we could have said, it could, it's mapping from marriage problem into a matching ok?

And stable matching mechanism is a mechanism that produces a matching that is stable for the underlying preferences ok?

Give me any preferences, the stable mechanism or find you a matching that is stable relative to those preferences ok?

A mechanism is set to be, ah, strategy proof.

If it is a weak dominance strategy for each agent to report, his preference, here ordinal preferences choose free. Ok?

So, we could sometimes, ah, call a mechanism to be strategy

proof mechanism, if, ah, it implements the a lot of matching in the strategy proof fashion ok?

And here comes the impossibility result.

Ah, no stable matching mechanism is strategy proof that is no, ah, stable matching mechanism that is strategy proof.

Stable matching mechanism that is strategy proof ok?

Or another way to think about that is stability

and strategy proofness cannot be joint with satisfied ok?

And the proof is by way of a counter example ok?

So, here are, let's say, example due to [?08:16], two agents on both sides, here is two men and we have two women. ok?

Men's preferences are exactly like this and women's preferences are

like exactly kind of this way ok?

It's like a matching pairness game structure ok?

So, think of men coming in either one man has a hat the other man doesn't

have a hat ,women are also, one woman has a hat , the other woman doesn't have a hat ok?

Ah, um, man with a hat likes woman with a hat ok?

man without a hat prefers woman without a hat

woman with a hat prefers man without a hat

And woman without a hat prefers man with a hat

That's the , sort of pairness structure.ok?

Ah, and then , there are two stable matchings ok?

This is men optimal , this is women optimal as stable matchings

Basically, men optimal , man gets their, his way

Women optimal gets her way ok?

Ok, ah, so, in this environment , if our matching mechanism is stable ,

it's a stable mathcing mechanism we are talking about

Then, this is the only two stable matching , ok?

Ah, given the truth for preferences , ok, if everybody reports truthfully ,

let's say, everybody reports truthfully,

Then the matching mechanism must choose either way ,this or that ok?

[10:00]

Suppose it chooses that ok? And possiblility right?

And suppose that everybody except for woman 2(w_2) reports truthfully ok?

Now, to show that there is no stable matching mechanism choosing this outcome when everybody reports truthfully

Cannot be ,ah, the strategy proof is surprised to show that ah,

when everybody else reports truthfully , woman 2(w_2) ,ah, wishes to deviate and lie ok?

And submit a different preference list ok?

In particular, imagine that woman 2 reports instead w_2 , and w_1 alone

So, what's the difference?

m_1 and m_2 ok?

Now, what she is lying about is that fact that m_2 is, in fact, acceptable

but she is saying that m_2 is not acceptable ok?

Now, in this case, the only stable matching relative to this new preference profile

is this ok?

With woman 2 lying and everybody else is telling truthfully, telling the truth

This comes out to be the only stable matching ok?

Easiest way to see is just to re-work everything with this and

you will see this two other same and that equals that

That's the easiest, that sort of the [11:50 brute]-forced way to get it

Another way to get it is the following ok?

So, now once we remove this, number of possible, that's not fact, the number of possible,

it is easy to see that this tree remains a stable matching, how can you see?

Then well any, anything, any blocking pair, any pair may block this ok?

Exist even before, here, I am sorry, so, let's see

By removing, by eliminating choices ok? You can only reuse

the set of blocking pairs ok?

So, if there was a blocking pair with this, that blocks this, they should have been a blocking pair

that blocks this in the original case ok?

So, that's one argument we can use to argue that this remains

a stable matching when you, sort of, remove some men to be, ah, declare some men as unacceptable

So, anything that blocks this matching without this should have

block that with this ok.

So, by reducing the set of acceptable men, the simple observation is

that you can only reduce the set of possible blocking pairs ok?

Because blocking pairs mostly form only somebody that is acceptable

And second observation is that no other matching can be stable because

the only difference here is that you know, if this is not the case

Any stable matching must be individually rational, so if m_2 is not match with m_1 ,

and m_2 is declares unacceptable for her

The only other possible stable matching must be, must match w_2 or

to herself, right?

That's the only other individually rational way of matching. w_2

And it must be individually rational. Because any stable matching individually rational,

it must hold, ok?

But then, ah-ha, we have to recall [14:16] theorem ok?

it has its news after all, because we cannot possibly have a problem where,

ah, some woman is matched to one stable matching

And yet, is not matched another stable matching

So, you cannot have, you cannot be sure that,

cannot be any other matching is stable

Let's, more sort of logical involved way of concluding this must be only stable matching

, ah, with that lying we talked about, we thought about

Ah, but, I mean another way to do it is to run through men optimal

and women optimal and then conclude herself that this is the only

OK, so, suppose, so here is the initially how we start in case you forgot ok?

[15:00]

so, in case of everybody is telling the truth,

stable matching must choose either one of the two

What we suppose is, suppose stable matching pick this one ok?

What we found out is that if everybody tells the truth,

and w_2 tell a lie saying that m_2 is unacceptable, she can actually induce this matching

Why? Because the matching mechanism we are considering is stable matching mechanism

which means that in case there is unique stable matching that must be the only one that should be picked, ok?

So, in case of her lying, because the matching mechanism that's not know

when somebody lies or not, right?

So, the only thing that you base yourself in terms of determining

which matching you choose is the report ok?

So, with this lying, this is the unique stable matching,

therefore the stable matching must pick that one ok?

But remember the woman 2 is strictly better off in this matching

and in this matching, ok?

So, there is an incentive. Therefore, two for woman 2 lie in this way, ok?

now, we are not done yet, since there are two possible matching

the other possibility is that stable matching, in case of everybody telling the truth, picks that one.

now, you can use the same sort of argument, in case of everybody except for m_2 telling the truth, and 2 have still preference for declining w_1 unacceptable.

and sort of having this chosen, he can expect this matching to be chosen.

and as a result, he will be better off.

so this prove that, this is the example, there is no stable matching mechanism.

there is also strategic proof.

in fact, here is generalization, which follows exactly same argument, that I want you to look at the proof.

so you use this [17:27] observation there as well.

what is says is there in case there are when everybody tells truth, in case there are multiple stable matchings,

so if there in case there are more than one stable matchings, in any stable matching mechanism attrivan? one agent[?17:49] can profitably misrepresent his or her preferences assuming that everybody else tells the truth

this is sort of impossible result

so far the news is negetive, result is negetive.

here is a positive result.

the positive result thing, at least we can make sure the agent of one side has the good incentives

it is called, one sided strategy-proofness result,

and it says that and that good incentives can be deprived by the nothing other than the deffered acceptance algorithm.

now MPDA, man proposing deffered acceptance algorithm, makes it dominant strategic, weak dominant starategy for each man to stake its true preferences.

and of course, that means that, of course , inelegance result, we hold it for woman proposing deffered acceptance algorithm,

to prove this, one has to use the following lemma it is called blocking lemma.

going to hope to prove this and then you know before the end of todays class,

so it says the following, consider any arbitrary individually rational matching, μ with repect to strict preference with P

I mean of course for you to determine the maching is individual rational or not.

you must know your preferences.

so given any individual rational matching, μ , relative to P , preference P , let M' be the set of men who prefer μ_2 , μ_M . okay?

so this is a man optimistic stable matching, relative to that preference.

we fix the preference threw out by the way for this lemma

for the prove itself, we are not going to exactly there, but, you know, I will show you how to apply this lemma.

[20:00]

so since pick μ to be any arbitrary individual rational matching, could be that M' which is set of men, who prefer this over that, could be empty.

but supposing, it is not empty, then, that means that there are some men, in this set who likes M

over μ_M

now, of course, if that would be the case, somebody who likes this matching to that matching, none must be blocking pair.

So, let's say.

the claim says that is always pair and W that blocks μ

such that, and that, furthermore, we can find the man from this set.

the guy who blocks is not the like the guy who likes, that's obvious in some sense.

it has to be that blocking pair must come from, it took block μ ,

it is not actually obvious, I shouldn't say.

to block μ it must be the guy who doesn't like, who likes the original matching.

so that promise, this is minus, it is just a remainder, set minus.

and then we can find the part of men are from this set, and women from this set.

so what is set, this is the set of women that I am matched on the μ to those men who prefer M' over, who prefer μ over μ_M

[student question] 질문이 있는데요, 그럼 저기서요 μ 가 M' 이랑 W 에서는 stable 하다는 거네요

[professor] μ 가 M'

M' 이랑 W 에서 정의된, μ 랑, stable 한거 아닌가요

지금 μ 가 stable 하다는 얘기는 없었지, 그냥 individual rational 한것만 있었고, 그랬을때,

one thing that I forgot, for if this one is not empty, okay, then we know about μ cannot be stable

because we know that, if μ are stable, this set should be empty.

that cannot be any man who strictly prefer this μ matching.

so in fact, μ_M must not relate μ for man

so what we know is this must be blockable, this is unstable.

and individual rational, it require to be individual rational.

so therefore, the only way for it to be unstable is for the acting to be a blocking pair.

the fact that they should be blocking pair is not surprising.

it just follows naturally from what we know already.

what is new here is that the particular way, I mean the ability to construct blocking pair, in a very particular way that we can form a blocking pair by choosing a man in this set, who likes original matching.

so it's kind of the unhappy guy.

this from the unhappy at μ essentially

the switch from μ_M to μ disguising in the guys are become happier.

and the guys in here become unhappy.

so you can find one unhappy man

and the woman who is matched with happy man to block that μ .

what was your question was?

what was your question was something was stable, whether μ could be stable?

[student talking]

M' and, M' 이랑 w 랑,

[student talking]

that's a very good point because, I don't know actually, yeah, right

of course, no actually, relative to μ_M of course

[25:00]

it's quiet clear that everybody likes μ , relative to μ_M ,

but we are not quiet sure there could be another way of matching, that make some of them μ happier.

so it's not clear.

so I mean, we know that μ_M cannot be the blocking matching, matching that can be used to block μ that we don't know that there may be another matching.

Again, you can prove this,

but I am not going to go over the proof

I am going to go over the proof of an one-sided strategy proofness result.

so I move to first assumed the preference is strict

but I do emphasize here I do stress here that the result to go through even with weak preferences
so, start with given any, start fix with arbitrary profile preferences
so that's the arbitrary profiled preferences.

and let's say P_m is the true preference for man m .

now what about P_{notm} , doesn't matter, because what we are doing is to check [26:46 the
sortion] concept is dominant strategic [26:49 influence]

we don't care we want incentive to be truthful, to be right, regardless of what the other guys in
fault

okay? Just pretty knows, fix $P - m$ arbitrary, fix it arbitrary

now suppose for some $P-m$, man M wish to lie and report something other than P_m

let's say P_m' is the other preference that you want to limit

now the mechanism you are using is what, man proposing deferred acceptance algorithm, okay

so when you lie, you are going to induce different matching clearly.

and that is man optimal stable matching(MOSM) relative to new profile

okay? Because you lie.

and such that, given that this guy wants to lie, because I am going to eventually, come up with the
contradiction of course.

but I am saying, starting with suppose otherwise, suppose there is the guy who wants to lie even
though his preference is P_m , he wants to lie by saying reporting P_m'

and that means that the fact he was lying, it was better as a result of lying. Okay?

such that this is true, when we talk about preference, preference ordering, we have to be very
careful about under what preferences is the case.

when I make the statement, I am saying that this is true, relative to true preference P , rather than
 P_m'

of course, I am relative to P_m' , this guy this is not true, false, because you are choosing man
optimal stable matching.

well actually I am not quite sure because this need not be stable, either.

but in anyway, the point is that supposing one wants to lie, that means this, relative truthful
matching, truthful preference.

then, this new matching, new, you have to first, I mean when you want to prove something, you

got to have a good sense of the direction.

what you want, where you want to go, okay?

in particular, what I mentioned is, we are going to use the blocking lemma to prove this.

so what is condition needed for blocking lemma to be applied?

while this μ , alternative matching, which is the matching that you induce, as a result of lying, you have to be shown that's individually rational relative to P , not relative to P' okay?

because, from the perspective of this blocking lemma, we are talking only, we are fix in the preference

we fix the preference and comparing the two this matching, you know man optimal stable matching.

and an alternative matching, that is individual rational, relative to the P

[30:00]

now this guys same, here, okay this is a man optimal stable matching relative to P .

this is man optimal stable matching relative to that guy that you get, as a result of lying.

the question is since the true preference is P , the question is whether this guy is individually rational, if he want to use blocking lemma, that has be the case.

now, it has to be individually rational, why because every agent other than M is the same right?

so therefore, if this matching, here is stable matching, which it is relative to P' must be individual rational relative to P - m which is the same .

so it must be individual rational relative to P - m , which is the same, so it must be individual rational for all agent other than M , that's one thing.

but what about M ?

for M ? that argument doesn't work, because it's different

but no worry because, we are really rational on this. Right?

But no worry. Because we are little assume this. Right? Because this guy by lying is better off relative to is true preference. We know new individually rational on the p .

Now we can use blocking lemma that exist M prime and w prime that blocks μ .

Such that M prime can be found from this unhappy guy.

when you switch from UM to U , there is an un happy guy.

and the woman who is next with the guy who is become happier.

so we know that. so the fact the prime can be found from this. It means the prime cannot be the same as M .

Because M becomes happier.

W prime also blocks μ . We know that we can find a pair in this lay in blocks μ

This is actually μ , μ is right relative to P . This basically shows μ is unstable because the blocks given P .

What I'm saying is this guy here also blocks μ given P' .

Because M' guy is different from M .

And now U from preference P to P' the only preference for preference has changed.

None of this guys preference are changed.

If this pair block U relative to P , this guys must block μ relative to P prime because their preferences are same.

That's a contradiction, that means μ is not stable relative to P prime.

How did you get μ ? μ is the matching. arising from the application of man proposing deferred acceptance algorithm

Arising from man optimized stable mechanism. This mechanism always produces stable matching

hypothesized μ matching comes out of it has to be stable and shown that it cannot be.

Which suggest that that cannot be a man who better off by lying.

The only strictly thing is you have to keep track a what preference when you talking about.

what preference is talking about for each statement,

[35:00]

so that is the only thing, here in some sense, I am not very precise here.

so when you say, blocking, I said actually P , if there is a pair that blocks μ on the P , that's given by the blocking lemma,

this last statement is given by the observation, that the preference of this blocking pair have not been changed,

okay? So this is the result of blocking lemma, this is not.

and then you have to write down here also, relative to P'

[student talking]

so we are saying is that so lemma, blocking lemma basically suggest following

you can always find the blocking pair in this way the man from this set, woman from this set,

okay? Now, what is M' here in this case?

M' is the settled man who would prefer better off, by switching from μ_M to μ .

so in another word that in fact that you find M' , there is a blocking pair consisting of men from this set, it has to be an unhappy guy.

by assumption, by hypothesis, M is not unhappy, M is happier moving from this to there,

so if you pick the blocking pair in this way, which you can by the blocking lemma, that man cannot be this man.

because that men are unhappy men, this is happy man.

there are two comments, and I will be done.

sort of about running over time,

the statement so far is based on the assumption that the preference is strict, but it is in fact true, even for weak preferences.

in fact the statement here, doesn't assume, the preference to be strict.

the weak preferences in fact, if you are really careful, it doesn't really I mean the same argument can be used essentially.

so how do you do it?

so what happens in the event of the preference being weak?

let's say the preference are weak, then, some men may say that you know, I am indifferent over settled women.

that's the weak preferences

then what would you do when around MPDA, what I mean, indifferent class, we uses some arbitrary tie breaking rule.

we create, generate strict ranking for everyman, for everywoman as well.

that is consistent with the underlying ranking, that I submitted, which make include with preferences.

and then what we do, the argument follows exact same way, in the following way, in particular so start with any arbitrary tie breaker for original preference,

for the new preferences, involves lying for one agent, the argument can be that, we pick the tie breaker, that is most favorable for this guy, for M guy, most favorite [39:27] mu essentially.

in the event of tie breaking, mu is most likely to be chosen.

and then the same sort of argument applied, if there is no incentive to lying, when you actually stalk up the cards in the most favorable way, toward the lying outcome, clearly, I mean for any other tie breaking procedure

there is no incentive to lie.

so that's the roughly the argument.

[40:00]

and the last thing that I want to mention is the statement can be actually also strengthen to group strategy proofness.

what is the difference between the group strategy proofness and regular strategy proofness?

group strategy proofness simply means you cannot find any set of men, any sub sort of men, who will be strictly better off

any sub sort of men who will be all better off strictly by lying collectively.

okay? So here it is an unilateral defection to lie that is illimitate, okay?

here, we can say in fact result can be stronger, in other words that there is no group incentive to lie,

but the mean that as a result of lying, some men will be strictly better off, and others are weakly the same.

that is not what I am saying,

there is no way for substitute men to lie jointly and everybody doing strictly better off.

you cannot at least rule out the possibility.

and the argument it should be similar right?

because here, instead of considering, M' for one man, will be a substitute man,

and then the argument party is also similar here

because we can be sure that the blocking pair consist of men, that's not part of M'.

that's not the part of the co-relation of lying.

and therefore, the blocking lemma can be still use, to produce a contradiction.

so that proof can be found in the [42:03 law cencord mio] book

I simplified the proof to just prove in some sense the weaker result.

but the weaker result is the most important result.

one-sided strategy proofness because often you don't really worry about collusion.

so that's the standard notion of dominant strategy incentive comparability.

it is not group dominant strategy incentive comparability, right?

for that purpose, this proof actually works,

so this is slightly simpler than the proof you can see in the book.

but this is the one the new one in fact.

I hope that it was slightly easier to understand.

I mean all you do is to just apply lemma, and there is a one set of nifty move at the end but that's all.

so let me stop here.