Title: 시장설계이론1, 안정적인 매칭의 성질, 전략적 이슈 (1)

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- ✓ **Dictated: 강은경, 강성호, 김신희, 김종백**
- ✓

[00:00]

so what we have done, we are, do we, studying, um.. multified matching.

so that is uh.. on one side that is a set of man.

and on the other side that is set of a women and we just figured out different ways for man to be matched with, with women, one man for each women.

and particularly we are interested in stable way of matching man to women.

by stability women, women sort of robustness to a blocking.

so robustness to blocking by an individual agent.

so meaning that the matching must be to be stable individual relational in the sense that no agent should prefer um.. remaining unmatched to um.. Big matched to according to that given matching function.

and then we also robustness to blocking by a pair of agence man and women that are currently unmatched.

but they prefer to be matched with each other.

whether than to be matched according to the matching function, okey?

so those other two requirements, types of reqirements for stability.

and within we care about the stability.

it's on the one hand, we, I mean, I mean, weexpect the stability to be um..equilibrium in condition, in many different kinds of settings.

because if a matching is not stable we expect, you know, there to be some sort of defection.

people who think that, you know, who can do better and a set of reapproach to somebody who

can form a collisional.

and so therefore unstable matching is not likely to survive this kind of disequilibrating sort of maneuver.

and also we mention that it's a sort of, a sort of desirable future from a perspective of designing a matching mechanism.

because if the centralized matching, that not recommend the matching that is not stable.

that's not recommend that the stable.

then uh.. Why don't we expect there some sort of the expections along the same line that I discribed.

and last, I mean, we think that stability contain some flavorable fairness, okey?

and that notion will be come clearer as we get to sort of the last segment of the lectures, lecture series dealing with um.. School choice application.

so what we did so far, was that, you know, first of all, you know, that is a necessary question, right?

we don't know exactly what kind of, want to a matching problem..

we may deal woth, we may in counter with real world situations.

what kind, in terms of, you know, who the man are talking more with the women that we are talking about.

and we don't inparticular know, we don't inparticular know what kind of preferences is that they have

so when we pose that question of stability, I mean we want to have the stable matching.

we must be, first of all, assure of existence of stability, stable matching.

so that, that was the first question.

that's there exists a stable way of matching, man to women regardless of preferences that ,you know, the man may have and the preferences that women may have.

and the sort of impressive interesting answer to the question was given by gale and shapley in their classical paper.

saying that, you know, in fact, there does exist a stable mtching regardless of all those details, okey?

and the way that they prove quit um.. quit um.. Informative and stricking as well.

because they did actually prove not, not, do they not just uh.. Establish existence.

in fact, they establish the way or an algorithm to find a stable matching, okey?

the two different variance man proposing differed acceptance algorithm and women proposing differed accptance algorithm.

and then we have shown, first of all, that each either sort of procedure generates, I mean, terminated by finite time given that finance set of man and finance set of a women.

and then it produces a matching that is stable, okey?

these two different variance did not produce the same matching.

but both, I mean, even if they are different, they are all stable, okey?

so one thing that I also described and also it's the part of the problem said is that stable matching is also pareto optimal.

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matching is stable, it is also plat optimal as long as the preferences are strict, okey?

so the existence of stable matching doesn't depend on the preference to be strict, okey?

but the fact that, stable matching is pareto does depend on the strict preference.

so I can gice you a simple example.

so think about they're being two men, m1 and m2.

and m1's preference is and two women is indifferent.

okey.

so men2 matching, let's say a man1 likes,

oh I'm sorry, what am I doing?

so w1,w2, w1 and w2

so man2 likes women1, women2

and everybody else, let's say even women is indifferent.

let me see, let me try this, I haven't actually worked out my mind.

and then lte's see, if what's..

suppose..uh.. Think about a matching.

men1 is matched with women2.

men2 is matched with women1.

so…

okey.

so.. um..

the two questions,

is this matching stable, first of all..

two things to check is this individual rational.

the answer is.. yes, okey?

because I'm taking a convention here adopting a convention whereby

released only those that are acceptable.

okey?

so, okey.

so therefore they are all individual relational.

now, second question is that..

Is this robust to blocking by a pair?

Is it blocked by a pair?

can you come up with a pair of man and women that are not currently matched and, and do better, okey?

by being matched woth each other.

when we say, what is important here, that you should remember is that, when you say do better women in terms of strict preferences.

just recall the definition that we wrote down.

I'm not gonna go back and forth.

but recall the definition, they are remember that when we say um.. pair blocks of matching.

that means that they'll do better strictly by matchiung each other rather than being matched, being matched according to the oriented matching.

okey?

so and question is..

there is no blocking pair.

I mean the answer is there is no blocking pair.

so therefore there is only one person can do, can do strictly better from alternative matching.

In particular that's individual m2, men2, okey?

so the matching here is currently stable.

what I.. And second question is..

Is this pareto optimal?

uh.. The answer is..

is this matching pareto optimal?

the answer is yes?

so what's the definition of pareto optimal?

matching or any allocation.

matching is an example of allocation.

would be pareto optimal if there is no other allocaion that pareto dominates that allocation.

an by pareto dominating womenthat we can everybody weakly better and somebody strictly better, okey?

the question then goes now to whether there is another way of matching that the agence makes everybody weakly better, somebody strictly better.

and,

this is actually, your answer is absolutely right.

it's strictly, this is actually a pareto optimal.

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I didn't it was actually, right.

okey.

so, so here is more interesting example.

okey?

so the other example are, they coincided, in fact, stable with stability, stable matching are stable

and also it was pareto optimal.

so what about this?

okey?

so stability, there is no question about it.

because there only one person who has strict preferences.

so there is no way to create the blocking pair just as before.

okey?

but what about pareto optimality?

it's not pareto optimal.

because why, because that isn't alternative way of matching them, the other is the, the other matching.

okey?

whereby no body is actually, everybody, and so difference only now is that..

so now that is another matching which is the previous matching that I described, okey?

that's exactly same as before in terms of welfare for everybody except for that guy.

because everybody else indifferent.

they don't care who'll be matched with.

okey?

but this guy here prefers to matched with w1 rather than to w2.

and so.. What happens that?

you know, then he actually will be strictly better off.

so this example suggests that missmatching were stable is not pareto optimal, okey?

so the statement that I made actually matters, I mean, really relys on the assumptions of the preference are strict, okey?

so preferences are strict for everybody.

then uh.. stable matching is necessarily pareto optimal, okey?

but not if the preferences are not strict,

okey.

so and then that's one thing you did, okey?

so we establish that there does exist stable matching.

and the second thing that we did show was that um.. We have two ways of finding stable matching.

so man proposing stable, man proposing differed acceptence and women proposing differed acceptance.

so two algorithm, so um and they identify two stable matchings.

they may be some other stable matchings that are not identified by these two methods.

and yet we can, and then these two matchings that are identified by this differed acceptance algorithm are interesting of special interst in part.

because it's extrem in terms of welfare, okey.

so it turns out which is a second thorem that we proved.

it turns out that the matching generated by man proposing differed acceptance algorithm is what we recommend optimal stable matching, okey?

so among the stable matchings, it gives the best partner for man.

At less? Weakly for everybody and may be possible strictly, okey?

so the man proposing differed acceptance algorithm gives a matching that is most prefered to man among the entire set of stable matchings.

it gives you the best matching for among the set of stable matchings ok?

so now the other seem that is important also is the assumption that the preferences are strict because we are when through an example ok which was last thing that i did in the class um that um if preferences need not district then that mean they may not exist a single matching that is best for man among the set of stable matchings ok?

you may be in a situation where are two kinds of undominated matching from the perspective of man within the set of stable matchings but they are not through ranked each other in the sense that you know some men are better off with one matching than the others and others are better off the other matching than the others ok?

so that's the seem that we this style discussed

this part here the thorem which called weak pareto optimality for the proposing side it sort of pursue the same sort of question ok ?

asked the same through question namely the we know that the matching generates, come out of

men proposing stable man proposing deffered acceptance algorism is best for men among the set of stable matchings

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now if you remove these some qualified among stable matching ok ?

is it still true i mean, another word that can you say more along this line in other words that can there be other matching not necessarily stable that can dominate the matching they say men optimized stable matching ok ?

we know that there is no other stable matching that does the best better dominates this but is there another matching that can dominate this matching

the answer is kind of some of new ones but it's true in a certain sense again assumes two preferences and what it says that even if you restrict the attention to very large class of matchings ok ?

um meaning that just individually rational matchings ok there is no matching there is individually rational that's strictly that are old men ok?

then the men of optimistic stable matching

if you sort of widen the cast widen the net if you will and then loop for bigger set of matchings namely any matching that is individually rational that doesn't dominate strictly for men in comparison with the men of the domestic stable matching

so let me just go through the proof it's not old that unimportant result let me just underneath proof because i think that it's useful to get used to a lot of you know arguments along this line ok?

so fix any consider any we already know this is already well defined this is the men optimized stable matching ok ?

consider any any individually irrational matching need not be necessarily stable ok? but individually individually irrational ok ?

and assumed ok actual this natural suppose so we are going to prove we are going to try to prove ok? suppose otherwise suppose that is a matching that is individually irrational and do strictly better and strictly prefer by all men over the that guy

suppose they exist in individually rational matching such that it gives better matching better mate than old men ok? than old man ok?

so our hope is to come up with some sort of contradiction at the end ok ?

so this is like supposing not supposing not by a contradiction that is supposed individually rational matching that gives better mate tricky better mate for every man ok?

then so what we know what i mean because we know that this is also stable what's be individually irrational

what it means is that this is true despite strict preferences assuming strict preferences sometimes

you guy write down weak inequility because what this means is that idle you are matched to somebody you stick to prefer to being unmatched or you do remain unmatched in this matching ok?

so that's what it means when run equality ok?

so but then the fact that you do actually strictly better means that in this alternative matching everybody is matched every man is matched right?

ok

so here I mean this is sort of very sneaky way of implying that you know it's now it's a way of fact that you know this is strictly better simply means that everybody should be matched in this alternative every man should be matched in this alternative matching ok.

now let's imagine that so for each man let's think about this a woman ok? that's he prefers strictly to her mate his mate on the man opimal stable matching ok

let's imagine now that you're run man proposing deffered acceptance algorism now we know the end outcome the end outcome is that every man settles with this mate ok? given by this ok?

but before reaching her we also know that given that each man proposing in the order of preferences and in terms of his preference this woman is out of the finite match so what we know this is that must be that he actually approached before i mean this woman before and got rejected ok ?

so everybody everyone among this matched to here ok ? let's think about this woman here ok ?

received proposal and now she rejected him and not because he is unacceptable we know that he is acceptable why because this was stable meaning individually irrational ok ?

so actually let's think about that so that was not that i was incorrect actually right?

this assumption was new individually irrational so that not individually irrational not because what i said ok?

so let's go back so he approached this woman let's call w prime ok ?

that could only be because then that is another better opportunity better alternative right?

so every woman that is matched ok? must reject him in favor of some other men ok ?

so what, that means remember this special feature of the deffered acceptance algorism

why don't you reject somebody in favor of somebody else we are guaranteed to match to somebody else at the end of day because for women you can only get better ok? remember that

so therefore any woman that is matched under this alternative matching ok? must be also must be matched eventually remember? ok?

under deffered acceptance algorism so under non-proposing D A[NPDA] only woman let's introduce a new notation ok so what is this notation? this is a set of women ok?

this is a set of women that is matched to some men ok? under this matching view

be careful here if you want to be really precise this is exactly so ok. exactly this that exists some men such that ok?