# Title: 시장설계이론1, 결혼시장에서의 일대일매칭 (2)

- ✓ **Instructor: 최연구**
- ✓ **Institution: 연세대학교**
- ✓ **Dictated: 강은경, 강성호, 김신희, 김종백, 이지현**

[00:00]

but might have realize by now that what I am talking about is somewhat different from what you know to be true.

I mean all of these are big ~such matching, kind of associate with macro.

it is some connection there in the sense that you know the stability is important notion that's all it be the sort of literatures well

and but the way they approach such matching in the macro literature is they approach them with some equilibrium behavior.

try to think about really how to model worker searching for jobs and what can you say about equilibrium outcome, what are the application such matching behavior.

on the what conditions do we expect to see sort of super[?00:53] matching, [?00:55] matching better workers that match with more productive firm and so on.

in that sense that motivation is quite different we are more interested in design or *normaltive?* aspect of matching.

well macro search match is more interesting in positive theory.

we tracked that using building block which they use to talk about lot thing, unemployment and other things.

the other important difference is that they want to add more realism by introducing friction.

whereas even when we approach a stable matching, in a problem one to match is equilibrium notion, we don't have any notion of friction.

the friction is market.

every man is available to every woman and every woman is available to every man, which is typically not the case in the real world of course, which is why I think that equilibrium stability

increase the notion, not so match attractive in some sense.

and also, the man is suitable gain a sharply model of matching.

positive theory is not very realistic and appealing.

because we don't believe that every man is available to every woman and every woman is available to every man.

okay, so which , we can add [?02:20]. we can relax the [?02:21 *ception?]* by introducing some sort of frictions.

people randomly match then decide whether to finalize the matching on, consummate the match or not, that's the approach.

there is some connection there my original plan was to append my part of lecture with somebody with more familiar with macro search matching just to give you better overview of entire matching literature.

because search matching is very very important in marco.

but it turns out that the plan is more doubt.

so talk with just me for that reason.

so now that we have established one thing we need to think about here is the, it's there the two things I wanna talk about.

one is that [?03:09] describe the algorithm and something that involves man applying approaching woman and propose.

and it's very through algorithm and very descriptive flavor.

think about man in a some sort of bar right,

and there is a man in the one side and woman on the other side, somehow they act in a certain very concerted way, approaching woman and the woman decided to accept the reject and so on.

that, that just for metaphor, now for we to learn a particular default acceptance algorithm.

you don't need to have a man actually approach woman and apply, you know, propose that woman, none of this, right?

why because only need is a set of computer and then we just have man summit the rank order in rankings of women, and woman summit their ranking of men.

and then we can simply simulate this process.

this is a computer program, computer algorithm, okay?

so they don't have to approach woman and propose.

and woman do not have to through look at the man and decide, so none of these would be necessary.

so we often discuss algorithm in this way, but just for your own interest and convenience, what it really mean is sort of proxy version.

we call proxy version of the mechanism all that is involved in submitting your preference list.

and then computer is then use to execute the algorithm, as if actually they go through this process.

the other important process, issue is now it seems that there is no incentive problems here.

we really didn't worry about incentive problems.

[05:00]

when man proposed to the top choice in the algorithm, here we didn't really worry whether he actually has incentive to do so.

in other words that if you think of in terms of proxy version of the algorithm does each agent, men or women have the incentive to report the rank ordering truthfully, okay?

we didn't worry about that we almost pretended that each person submit ranking truthfully.

but then that's not problem, because that's not a problem from the prospect of theorem.

because the theorem said nothing about incentive problem, it simply said, that a stable matching exist.

the algorithm and the assumption that when participated in the algorithm agent actually for truthfully just an argument.

it's sort of tool of argument.

meaning that incentive issue is irrelevant for the performing.

cause, all we claimed this isn't exist.

this is a way, this algorithm simply used, to produce a matching,

and we proved that the matching is stable.

and therefore, we've showed that the existance of stable matching ,okay? for each marriage problem.

which is not to say, that incentive problem doesn’t exist, and it exist you worry about it in later.

but what we can , the claim is still true, another words, there is a stable matching, for each

marriage problem.

that's not issue.

just to get your, gain better understanding about how the mechanism, how the algorithm works, and the notion of stability itself,

it considerable, stop by asking now, now we have established existance of matching,

the next question is unique, okay?

can you expect to see a unique stable matching?

and the answer is not necessariliy,

okay here is example with multiple stable matching,

and here is also good opportunity to basically get familiarized with a deffered acceptance algorithm

so the way I would show, that there are multiple stable matching, I am going to by running this two different versions of the deffered acceptance algorithm.

and show that they two, these two produce different matching.

and both of the mis-stable. of course.

let's run the first man proposing the A .

so the way to do this, just to fix woman, this is the way I do,

and in the first round, look at man, who did liked the most

so one or two, like one, actually this case, everyone likes woman one, right?

so, okay?, and then we go back, to the woman who has received the multiple proposals, okay?

and everybody is acceptable, not can take only one, okay? .

Takes the only the best one

so two or three kicked out permenantly.

then, man one in the next round, man one has nothing to do.

here is a proposal already accepted, so he has nothing, he cannot do anything.

this two guys have the most proposed to next best, w2 and w3

so each woman [?9:20] now received a proposal, must decide whether to accept or reject.

now the only thing to worry about is whether the man 2 is acceptable for woman 2 and the man 3 is acceptable to woman 3.

now they have to accept them in that case.

that's it

no more rejection, the algorithm terminate in to step in this case.

we have pin down to matching, man 1 is matched to woman 1.

m 2 is matched to w 2.

and m3 is matched to w 3.

[10:00]

let's do [?10:02]

woman propose the a

we fix the receieving side man 1, 2, 3

and then one through this multiple step.

first we think about who the women'd like to propose to turns out man 1 is the the lucky one in this case.

and then now, man 1 is the only person who has received the multiple proposals, has to therefore decide, which one he wants to retain, he keeps a w1 okay?

w2 and w3 now be rejected by the top choise must propose to somebody else.

And 3 is next best to w2 and 2 is next to best w3.

so, okey?

And now go back to the receiving side.

and 2 finds w3 acceptable.

then 3 finds w2 acceptable.

again no more rejection

algorithm is terminated and we have matching, okey.

now, see that man1 is approach to, I mean, match to w.

because it is the first choice each other.

okey?

let's solve the unbreakable bone if you'll.. They'll be matched always together in any stable matching.

but the remaining agents are matched differently.

okey?

in two different matches

So m2 is w2, m2 is matched to w2, m3 is matched to w3, here.

but differently.

okey?

so and then this theorem actually applies to both verision, both variance of the Deproductism algorithm., meaning that they both is stable.

so, as you see, for reason we'll hopfully make clear later.

that um.. If both [?12:26 barriers] previews the same matching, then you'll have a unique matching, unique stable matching.

okey?

why that is so, we'll hopfully make clear later.

now, the other thing to know to say, at this point, is that, that may be another, some other stable matching which is not attained by either, either algorithm, okey?

so this algorithm, it can be used to reduce some stable matching but not all, okey?

he os an example where you haver, you have unique stable matching, okey?

one thing that is to be curious is that uh.. At less, I mean, an example is already slighted in front of what you have in the, in the handout.

because I thought that, uh.. It might be some of missleading of the example, they might be, might be a missleading.

for need to get this unique matching actually what tis important here, this not also, this not also um.. Mecessary for to get unique stable matching.

but one sufficient conditions here is that one side here, in this case, man side have the same ranking.

okey? Of, of women.

so, in his case, um.. Women 1 is top choise for every men.

so she basically calls shalpe.

she decide to who match to her. Okey?

and then one thing you've need that the women along with that man, okey?

most prefered man for her, okey?

and the next best person is, w2 next popular women is then she picks the best among the reamining men.

so that kind of how we'll work out.

but wou see, saying that sense that, that's be going to be unique matching.

so you the second problem, in the problem set basically tries to have the argue that…

in fact, that's going to be, that's going to be the matching, stable, unique stable matching in the situation like that.

and prove them..

okey.

so what then you can run through, you can run through the deffered acceptance algorithm right here in this case.

so let's do the man and woems side " MPDA", w1,w2,w3, first round everybody likes women 1.

[15:00]

okey. And she takes m1 rejects m2, and m3.

and m2 and m3 being rejected by this women

now go out tho the second most popular women.

okey?

and she takes m2 and then reject 3, m3 now reject w3 next step and m3 is acceptable to her.

so that's the [?15:36] matching ok?

and we are proposing here now it's a little bit different like wr and w3 apply to propose to m 1 (student speaking) I am sorry did I make mistakes so maybe so everybody likes woman and then and this 2 rejected m2 and then m2 likes this 2 and then

I see so you are right and 2 w 2 likes m3 right? Of course sorry m3 and actually I did ya so and sorry you're right and then m2 now propose to man 3 and then[?16:55] thank you

ok now this w1 and w3 propose to m1 right? And then w2 propose to m3 ok?

m1 likes w1 w3 then what happens 2 like m2 ok so here is the matching ok?

so it's same matching, you see?

so the rotation for the man proposing the matching generated by man proposing differd acceptance algorism usually you know subscript use subscript capital M ok?

and for woman proposing deffered acceptance matching we get used it subscript w.

ok so and then one thing that I didn't really say um so let me go back to the case with multiple stable matchings ok?

now what can you say in terms of the welfare for this [?18:11] between the two ok?

now if you are the man ok? is there agreement in terms of which matching is better? I mean there is no reason to expect that they will be agreement

in particular, if every man likes same woman in general their preferences are conflicted there is no real concensus in terms of which woman should match which man right?

but here that is not the question, the question is that can they be an agreement on the side of man within stable matchings ok? not all matchings ok?

and it turns out surprising in this case there's an example there is an agreement why because now in this case in man proposing differd acceptance acceptance algorism generated matching

man 1 is of course assigned to the first best that's best for him

m2 and m3 are assigned to the second best choices ok ?

where in the woman proposing deffered acceptance it's a same here but m2 is assigned to wc, m3 is assigned to w2 this all best not choice ok?

so every man is weekly preferred weekly preferred this matching over that matching some strictly so ok?

now look at the woman side ok?

in fact the opposite true woman likes this matching better than that matching ok?

so I told you before that you know it sounds as if their algorism woman sort of course is sharp right? in the man proposing algorithm they are the ones who this side who gets kicked out get accepted temperately.

[20:00]

how about in fact you know that's a better deal for man, if you want to choose between the two right?

Proposing ,ah, actually better , better deal

Um, now, but this property here, the fact that there is an agreement , is not ,um, is not achievable property.

In some senses , kind of , we want , would not generate , expect

that such a property to exist , to hold

And , it turns out that little ,ah, you want , can show that it holds

But , before talking about that , I mean , here is a little bit of patience , OK?

We say, you, this , ah, is preference inequality , ah , but ,

with a subscript capital m

In the set of all men here.

So, when we say , ah , this is ture , that means that matching m ,

mu( μ) is at least weekly prefer matching μ'( mu prime) for all men , for every man

and some man, it must , the preference must be strict . OK

Now, if you assume preference to be strict for everybody ,

what that mean is that , if , talking about this holes , that means that some men are matched to same partners between the two.

And, some men are matched to different partners, in each case ,

those guys are matched to different partners across these two different matchings ,

Ah, they are assigned to better partners, here, and here , OK?

That's what I mean

So, you see , whenever this is ture , the opposite would be the case

at least within the set of stable matchings.

But, we haven't gotten there yet.

So, let me just , um , establish the next important vision

We say that the stable matching μ is men optimal , so definition

Ah, if this is ture, another reason is that ,um, if μ dominates or other ,

stable matching μ'

So, among the set of stable matchings, μ is the best

from the perspective men, OK?

So, when that is the case , we say that the best stable matching

μ is men optimal

Sometimes, we say , ah , μ is men optimal stable matching

So, use this acronym, Men Optimal Stable Mathcing(MOSM) . OK?

And,. Likewise, for women. OK?

So, that's the definition , here is the claim, OK?

If preferences are strict , then there exists a men optimal stable matching ,

and it is generated by men-proposing Deferred Acceptance algorithm , OK?

Ah, and, there is a women optimal stable matching

and it is produced by women-proposing Deferred Acceptance algorithm.

Again, ah, this is sort of suprising result

You can , ah, there is a , among the set of stable matchings ,

that is a matching , that is best for entirement

there is an agreement on the side of men , OK?

Here is the proof , [?23:15] quickly ,so, to prove it ,ah, it is useful to define the term

So, we say that , ah, we use this

So, we say, we just move this ,

That says this pair of men and W are achievable . OK?

If , to each other , if there exist a stable matching ,

the pair of them together , OK?

So, priciple for , for men , for women , is acheivable to him

If there exist stable matcing such that the man is matched to woman

That can be multiple stable matchings as long as there is one stable matching

Under which , this , two pair are matched to each other

Then , we say that this two pair , this pair of men and women

are achievable to each other . OK?

So, just the definition , nothing more . OK?

What you are going to prove is that , ah,

if you learn the men-proposing deferred acceptance algorithm which is chik prefereces.

Ah, no man is ever rejected by unachievable woman

Since man proposes , ah , in the order of his preference . OK?

What this means , once we show this , what this will imply is that

she will be ,he will be, ah, match with the best achievable woman.

meaning that the matching gererated by the man proposing deffered acceptance algorithm.

[25:00]

it's, man is matched to be the best women that he can ever be matched with any stable matching.

okey?

which'll be the proof.

we'll do after.

okey. We need to show that nat that's the plan.

that's the ptoof is kind of very interesting.

That's the proof is kind of very interesting, is the algorithm oriented proof.

uh…we'll some of this proof techniques.

as we go along.

but uh, suppose that, to prove this, that no men is at all rejected by a woemn that is achievable to him, okey?

Suppose that otherwise,

Suppose that's not the case.

The some men is rejected by a women that is achievable to him, okey?

Since everything is final that they will come a time if you run this man proposing this deffered acceptance algorithm.

there will be the first step that this have happened

So, let's say that um.. Let's say that, let's imagine that the man M is the first man rejected by an achievable women, okey?

I mean that will be multiple man who are rejected by an achievable women in this round.

But you know, pick one of them.

So suppose that claim is not true, then they'll come uh.. There will be a first step at which some men is rejected by an achievable women.

So, so that's the case.

Now, so, so M is the first to be rejected by an achivable women, W same, okey?

Now, why do you know about that is um…

So suppose this focus on that step, okey?

Is that man acceptable to that women?

First question

okey?

The answer should be..

uh.. must acceptable

How do you know that?

It has to be the definition what's ability.

How did you define achievability?

okey?

W is achievable to M.

Only because that exists some stable matchings.

added under which this to apaired.

okey?

But for matching to be stable.

It must be individually rational.

It must be acceptable to the, to the other.

okey?

So we know that even though this guy is rejected

okey?

he must be acceptable.

okey?

and okey?

but he is rejected

how coul it be?

What's the only way that the uh.. M is rejected by a women?

what is uh.. Achievable?

So only because she has a better proposer.

okey?

uh.. Only because that exist and M prime proposing, proposed to W.

okey?

So we can deduce step.

okey?

And let's think about M prime right here.

okey?

now

so..

so, on the other thing, that, that is important here.

Is that uh.. She might have actually, we don't know who propose that exactly, right?

one think that we know is that M prime must have proposed to W?

either this step were before

okey?

so, when you say rejected, it could be, because M as, had proposed end was accepted and

rejected at the step.

or the other where are

so M must, M propose to her at a given steps.

and then, uh.. Somebody else there.

I mean, she had either proposer for M prime thinks that or prior.

okey. Stage.

doesn't matter.

right?

why think that we know, however,

Is that M prime never been rejected by an achievable women before?

How do you know that?

you know that because we made this hypothesis.

because M is the first guy.

And this is the step.

because, you know, some men is rejected by an achievable women.

okey?

so, has never been rejected by W

so, what the needs is that uh.. The fact that she either has proposed here.

[30:00]

so, um.. Without this guy, this step means that uh..

To him, to this guy, this women is better than any achievable women.

because men approach women in the what they wrote in the preferences

okey?

so the fact that he has never been rejected by an achievable women.

In this step either, this is the best achievable women to him or may be unachievable.

may be later this women will kick out this guy.

okey?

but eithet way this women is at less weakly better than all othet achievable women for him.

okey?

Anyway, so that's the sort of side observation.

Now, let's go back.

and to the recall the notion of achievability.

The fact that we're assume that W is achievable to M.

but the needs is that they are exist [?31:04].

we are not, we are leaving the algorithm now.

okjey? So now the argument that I'm going to make has not been called the algorithm.

Now, we have just uh… remembering, we're going to remember thing that, you know, distrupt to each other.

the means that this stable matching, you know, such that.

right, okey?

um.. Now, let' make two observations here.

okey?

so, first of all, let's think about this two people here.

okey?

the fact that W rejected M um.. In favor of um.. M' means what, she like prime better than M.

okey?

Now, the side observation that I made means that uh.. Means what..

means that uh.. M' um.. Lkes W more than any achievable women at less weakly.

okey?

In fact, uh.. Yeah

so, actually test be strictly.

um.. Now, let' make two observations here.

right.

Anyway, other than the one possibility is that, weakly means that she is actually achievable, right?

but, in other case, I mean, anyway, she um.. M' likes her more than any other achievable women.

strictly that's for sure, okey?

either, W is unachievable and, and he like her the best more than unachievable or uh.. Achievable women or she is ahcievable.

oh, but she dominates all other achievable women.

okey?

so, now, what we know is that M' likes um.. This is to [?33:14]

he is matched to, under this, this matching.

okey?

How do you know that?

because we know this, because newest stable meaning that whoever is matched to, whoever M' is matched on the mu.

must be achievable.

that's the definition of achievability.

okey?

so, um.. You made observation, okey?

W beats out or other achievable women.

other than W

okey?

so, what have we shown, we have shown that M' and W blocks mu which is contradiction to the claim to the hypothesis that the mu's statement.

so we studied out with a hypothesis saying that this could happen.

okey?

Another is that, you know, man can be rejected in the man proposing before the acceptance of this algorithm by women that is achievable to him.

and then, we constructed um.. Contradiction.

okey?

so out of this subtle,, but, you know, it's not a big deal as you know.

Spend some time thinking about it.

So, unfortunately, I, let me just uh.. Let me just go through this example and, and the end of today's class.

okey, so uh..so, Is that assumption, so ont think that we assume..

[35:00]

By the way, ask me if you are not clear uh… I think that, I noticed that may be this is a little bit subtle and little bit [?35:10] but yeah um..

If you run through, the proof, yeah, right?

okey?

I mean it, when I first solved the proof, it was very strange argument, it thought of uses how the argument algorithm works.

and make uh.. You know, so deduce from how the algorithm works to come up with the right, uh.. Right argument.

That's very very nice argument nonsense.

So uh.. What about, Is that assumption needed, yeah, that assumption, that preferences are strict.

is used, is that, is that, is that needed, in other words, if the preference is not strict then can we still have a guarantee existence.

so man optimized matchings the answers are turns out not to be the case.

so blanks are used, the indicate that, uh indicate that the preference, the mad is different between this two.

okey?

so it takes us one person is to be indifferent between two pairs to generate the, an example.

uh.. The shows that uh man optimize the stable matching may not exist and doesn't exist in this case.

so how do we do that here?

so we are going to run man optimize the different acceptance an algorithm uh.. Twice.

uh.. By using different tie breaking procedure, okey?

So one thing that I told you uh.. To run a deffered acceptance algorithm.

In case, the preference is not strict.

So they can, they are welcome, allow to submit nonstrict preference.

but then we use some way to, we have to break a tie.

so you needs to ranking this algorithm, okey?

that's the how the algorithm is structure, the only thing.

so therefore what you nedd to do is, in this case, you just arbirarily break a tie.

two different ways to brak this tie's course.

you can keep this ranking, W2 ahead of W3 that is one ranking.

Another ranking is, another tie breaking, uh, another way to break a tie, uh.. The other where are W3 ahead of W2, okey?

If you do that, so you can turn two different versions of deffered acceptance algorithm.

In this case, uh..you get one here and you get the other here.

And so since I don't have time, I'm not going to go through this.

but you can again go through the algorithm.

And, and confirm yourself these out of two.

What's interesting here is that, they are not rankable.

that is no consensus among men about which one they prefer as a whole, okey?

so, so , for instance, what is that, um.. M1 is, right, M1 is assign to his first best either way.

because he is indifferent between W2 and W3.

That's fine.

let's think about the other guys.

M2 is assign to M, W1, in one case.

uh.. And M3 is assign to W3.

M3 is assign to W3

okey?

that's what I discussed in the first matching.

In the second matching, M2 is assign to, M2 is assign to W2.

and, and M3 is assign to W1.

so, in one matching, this is happens.

In the other matching, that is happen.

okey?

So, in other words, these two stable matching treat them differently.

uh.. This two people differently.

Even though both are generated by the uh..man proposing deffered acceptance algorithm.

So, in this case, thete is no stable matching.

So this is the best you can get for the man.

compared with the all other stable matching.

okey?

these are two best.

um.. But there is nothing, there is no stable matching that dominates everything, everything else.

So you can at less possibly say that uniformly dominates that for all men, okey?

So that is conflictly interest, that is disagreement between the two.

And you get that only because of the difference.

[40:00]

So the stability argument.

In fact, I was careless in the sense that..

When I, um.. Prove the first theorem existence for stable matching.

When I show, I mean the kind of assume, the preferences are genuine restrict.

But you don't need that assumption.

the same argument tolds, showing that, you know, there is no blocking pair even with a non strict preferences that may be some indifferences.

that still follow.

okey?

but the course that I need indifferences that you have to rely on tie breaking.

but no matter how we break ties, but that means may be you make um.. Many different multiple stable matching.

generated by multiple matchings, generated by…

the same men proposing deffered acceptance argorithm depending on how you break tie.

bur regardless of how you break the tie,

the matching you get at the end of the day is um.. Stable.

the same argument is exactly [?holds41:10].

okey?

okey, so that's it for today.

so we'll meet you on Friday.

I'm not be proving the everything I noticed.

I take too much time.

but um.. I'm prove, I know, I guess.

at less um..

so we will do um..

next time, we are going to talk optimality.

some more properties of stable matching basically, okey?

And, and then we'll talk about incentive issues.

교수님 아까전에 men optimal 이랑 women optimal 이랑 일치하는 경우가 있을때는요, 그러면 stable matching 이 전체적으로 한 개밖에..

it is very good observation actually so um… because um.. Right.

suppose that.. Noticed that there is no reason to, there is no proof so far.

so just think that the all stable matchings are generated by two different [~]of stable matching.

so the fact that.. Uh.. We have established man optimality, women optimality.

that's not mean necessarily that you need unique stable matching when this to coincide.

okey?

but you can easily argue that..

because..

for only Suppose that mu stable for any stable matching mu, okey?

we know that this has to be the case.

so what does she say?

this is the men optimality.

the matching that you get by running the man's proposing deffered acceptance algorithm.

gives you a matching that is uniformily better for all men than any stable matching.

because that's, that's what he said, right?

better than all for men than all other stable matching.

okey?

Likewise, for the same is true but something that actually I didn't show yet.

So we are not yet, I guess ready to claim this

uh.. Is that.

this , this part is actually missing side.

Let me take back.

So we can not yet cnclude what I said.

So, but what is turned out to be true is this.

okey?

so that is, that is, this uh.. Oppositon og interest

whatever is good for man is not so good for women.

that is conflict interest between two sides

No conflict interst within each side

at least, If you, as long as we focuse on the set of stable matchings.

okey?

but this would turned out to be true.

Is the man worst stable matching.

man optimize stable matching is women worst stable matching.

that's the ranking at this anmong the stable matchings.

And the fact that this two other same means that they ~