Title: 시장설계이론1, 결혼시장에서의 일대일매칭 (1)

- ✓ **Instructor: 최연구**
- ✓ **Institution: 연세대학교**
- ✓ **Dictated: 강은경, 강성호, 김신희, 김종백, 이지현**

[00:00]

This is a course; by the way, I am hoping that it's ok for me to speak in English

I just don't care and I am Korean I just so that some of you are interested in going abroad to do ,in advance, research.

so I think that, you know, it may be useful for you to gain more exposure to English speaking lectures.

But let me set the ground rule which is that in you are welcome to questions whatever language that I can't understand which includes for some Korean and English.

So I will let's see how it go.

I mean all idea require the most is that high communicate well with you in a waited you can understand.

So, if it gets more difficult for us to effectively communicate, then we can switch back to back to Korean.

So, what I will be doing in next eight lectures including today is to give you a basic introduction into sort of fields, norm in economic matching theories

So, matching series are through one of the math logical frame work along with this use along with Mackenzie's design in area called market design.

Market design is been arising field in recent years that has been interesting and has gain a lot of tensions because ,you know, you has been used to many practical application in a way that was quite helpful.

So 'matching theory' is particular and very effective tool with market design.

Its specially, in understanding how professional labor market works, that is, you know, firms side, ,employers side.. and employees side,

the market open is not continues beholding through, that is, basic in a period time where there is market open and so the firms are and workers are trying to match other.

So I think that the understanding how thins matching come please out, how the matching please out this market is kind of interesting.

This is the useful in understanding that, but more importantly if you are interested in market design what you will be, potential interested in designing.

These matching markets do work better.

So, not only do we just want to use theory understand how the market works in labor market.

But, perhaps you want to use our understanding to improve the way in which this kinds of markets work.

So they are already several successful applications of matching theory in particular in medical sort of [3:31-32]applications mathematical matching

so they organized market through which doctors often graduated are medical schools to get assigned in hospital to work as a residents and interns.

This happens in this country and the u.s. and any place you can imagine and this is another very big coordinated matching in problem.

So, like many other places market was run in decentralized fashions, in a without sort of trying to, sort of creative some central clearing how to coordinate a matching in the beginning but then they ran into lots of problem and hopefully have more chances in the future through course to talk more about some of this episode.

But basic story was that market was originally organized in decentralized fashions.

But after running into lot of problems they tried to coordinate the matching better and ,in the process, economics play in the important rule and matching theories are be very helpful in that effort.

The other important applications are problem known as school choice.

I mean many of you've already heard of it.

Many countries in public schools from kindergarten level to high school level basically.

[05:00]

The six schools are basically provided as for free education to people but then I mean, schools are very in quality and locations, it is not clear which student's should seat attend which public schools.

The traditional way of assigning students to schools was by residence so the school, I mean, students tended to be assigned to schools are in the area which is probably student free dominant of assigning student to schools but then, simply means that you may be limited to very small number of schools in your neighborhood which maynot allowed for whatever regions.

And more different in social problem perspective is that we know that we know that some schools are better than another school.

Therefore, if the way to get in to the schools, get send you school, the kids to go those schools is to living that area.

You will be willing to pay a lot of money to relocate for a school to be cheap. Okay?

Which is true in this country, which has been is also true in the United States as well.

So, at that, means that if you are wealthy enough to send your kids to move to good schools in this strict, you will do.

If you are not so wealthy, then you don't have these kinds of choices.

Not only is the choice limited, within this system, and the choices are not equal across different people and another way to think about it is that essentially the housing market's act in some market for school essentially right?

There is no tuition, school is not prized, explicitly, but its price is implicated through the housing premium. Right?

So, in the sense that, I mean, sometimes goes against the idea of free public education.

Relying on essential the market, for assist to determine who get assigns to which schools.

So that, I mean, this sort of problems let to the initiative known as school choice whereby basically essencily set of expected school choice, beyond the liking the resident's areas and then its good idea, I mean theory, to give more freedom and you expand the set of choices available to students.

but then the question is really how to implement the choice. It is basically solving, matching problem.

Because, you will incorporate, you will not into simply to ramdomly assign them. Your right?

I mean, using in drawing lottery number for students because you will be preservable to take in consideration of the preferences the students have.

They may have preferences are different and then if you can, you may want to assign them in a way accommodate their preferences.

Anyway, those are serving to examples that are kid exchange example, my plan is not to really

Although I mean it is useful to give you a lot of good motivation, motivation goes a long way to create lasting interest which is very important.

At the same time, I think that it is very important to give you mythological frame or effective precise a way.

So I don't want to spend too much time motivating.

my plan, I initially want it to cover a lot of materials but then I realize it makes much more sense to limit the number of material to discuss.

But each material I spends enough time to give you good enough, through enough introductions so that you get something out of this course.

I hope that you stay with me and feel like that is something that you take away by the end of the whole lectures series.

So, in terms of the planning, I am going to focus on three different ingredients, relevant for the syllabus for those seen in the syllabus

The first part, that is know I may just use this.

Let me just use this.

white chalk will be actually helpful because I want to use some points in the blackboard.

[10:00]

The mythological is there are two different types of problems that is something call 'two-sided matching' so, all these problems…

So I mean it is like example that I describe.

That is a one side could be students .

Let me just call it, actually, workers because I want to start with a two-sided matching and then is a firm.

So the basic structure with two sided matching which I am going to more former introduce to spread on.

Instead, there will be some sort of matching workers are matched to firms workers are matched to another worker.

So that I agent both are the idea basically is that the structure is such that.

That you consider the divide the agency two groups.

One side of the worker are on the other of side worker, firms could be students on the one side and schools are on the other side, doctors are on one side and the hospitals are in one side.

So and then, the point is that typical location that we care about is how workers are matched to firms openly one to one on basis.

That is the basic framework, we gonna set of reach the structure to allow for many for the matching because in a firm may employ more than one worker.

But that solve basic two sides matching structures.

So problem where by agent on one side are matched to agent on another side.

The important thing here is that agent both sides are strategic players.

So… humans are on different sides. That's sort of the two sides matching; I am going to just use a term although I mean terminologists are not really well said.

Similar problem, similar problem, I will call it the 'assignment problem'.

In this problem, one side is individuals,economic agents, but the other sides are not a set of agents.

So the other side is objectives.

If you think of this as a school choice problem, student may be housing location problem that may be even better

The problem is to assign house, maybe apartment unit, to student one for each the student.

I mean problem in mathematical structure is very similar to side of matching but they are subtle differences, there are going to be subtle differences,

because when you talk about welfare and incentives, here we have to worry about agent in both side we have to care about the welfare of the agent of both side, we need to care about agent only one side.

These are two mains, sort of frame work we are gonna through think about the reason that I am dividing is that match of the structures rebuilding deals with incentive issues, incentive problem, as well as of theirwelfare.

I will do one application that's school choice.

This is application but also you can think of this as another, sort of more another framework just like this problems in the sense that in many public schools assignment problems.

The schools, this side is in unclear whether you should think of them as agent because they don't act strategically.

They are acting in a way that's sort of in principle, um, a shift to maximize the wealthy of students if you will.

But in some sense, they are acting strategic or act like agents.

In the sense, they othen have some priorities also care about the easiest way to think about it is that this is problem entering college or problem of entering some specialize to schools which care about student's academic abilities.

[15:00]

They administ exam test, some sort of entrance test. And they use this score from this entrance test in determining who get and who doesn't get in.

So, the test score could be a basis on each to prioritize to student.

There are other considerations that may play, are coming to play one of each could be how close the students to live to the schools.

If students are living close to the schools, then that may be welfare reasons to favor them because it could be better not just those student but also socially, because commuting becomes easier problem and doesn't really cause congestion in morning time, traffic jams, things like that.

So that maybe some externally some issues to worry about the social reason to prioritize the admission based on where they live.

And it could be also that one consideration could be similar to consideration whether or not, the African sister or brother have already enroll the particular schools then they are like to be in the same school in that they make it easier for the parents to do dropping off and picking up arrangement in respect of the kids.

I mean, of course, the range, they apply for reflected preferences in this regard, but then this preference may not be effectively communicate to in the process.

Anyway, I don't want to get in to the details at this point. So, that is some resemblerence us to matching their priority is setted, schools may set

You can think of them as preferences on a part of days in this side. But then, when you evaluate welfare, you can focus on welfare of the agent from the other sides.

There are going to be the main topics.

And so what are you doing the first a few lectures.

Just let's start with two sided matching, okay?

And, within two sided matching, one can, I am going to sort of thinking in terms of three different parts.

The first, we are going to start it slowly and gradually from the simple and possible problem, it's marriage problem.

It is another name for one to one matching.

Marriage problem because original sharply , so article that introduced this one-one matching problem Turned it that way

The introduce matching in problem are terms that way, okay?

This often terms are more mutual term, you know, it also comes with sum, so not the neutral.

The terms that is more, I say, geer to particular application that the people who invent this side here head in mind.

So, we are going to think about extending the results established for one to one matching to many to one matching problem from where typical application you can think of it is that of, you know, work for typical assignment or student assignment to schools are in the sense that one side can admit a number of students or a number of doctors and hospitals.

So, that is kind of situation. Okay.

And then uh when you introduce many to one matching, in particular, it becomes a little bit messy and through tricky to think about preferences on the side of the colleges

the agent on the side which admits or gets a sight of many agents on the other sides in the hospital doctor problem hospital admits matching of a number of doctors

and how do you describe the preferences on the part of the hospital I will talk briefly about that issues that arises

but it can be pretty complicated the preferences can be pretty complicated and many concepts need to be many basic through fundamental results may not go through

unless you, we put a reasonable restriction on the type of preferences that the agents on that side have

we are going to start with marriage problem and then do the extension into matching still we some simple restriction on the preferences of the college side

[20:00]

we are going to think about a basic model students getting assigned to colleges um and the college has some simple preferences known as response preferences

and then we are going to relax that assumption and what we'll do it to mainly focus on head fill milgram[?20:25] in terms of time and going to spend probably today talking about marriage model

and then perhaps this Friday I'll talk about college missions so it maybe by next Monday or Wednesday might go this to, get through the entire two sides of matching

two things. one I am going to take a break after an hour uh maybe 5 minutes

if we don't take a break and students actually told me that it's a better idea to take a break

um the other thing. I should have probably had you introduce yourself probably after the break let me do that ok?

um and i'm thinking that some of you are not from yeonsei(연세) right?

I got some emails from students who are from korea university and seoul national

I wonder if 9 o'clock class, make sense?

so if you I mean I've thought about actually asking you to see if you can move to us some other

day of time

thinking that they will like 6 people but since there are more maybe difficult coordinate which case

let's forget about it. but uh some issue in case that isn't need I think we should I am willing to think about this way

let's just get right to the business and start with the model so by the year I am going to try to go through the proofs of several fundamental results ok? Um

ok? um so let me just talk with the description of the model

so what are the main ingredients of the one to one matching

so there are three different ingredients

first description of players ok?

second is the preferences we model for each of these agents

third is the notion of matching ok? which will come in the next slide

let me just begin with the description of players

again the important thing is that the agents can be partition into two groups. with idea that the only matching we are going to be allowing is matching across these two partition ok?

not within agents and within each group

ok so there are specific names that are not important but we'll for the sake of a lecturing

let's say there are men on side there are women on the other side

there are finite number of men typical genetic rotation for each man is a low case same am i ok?

and then there is a finite set of women w, capital w again in the same location and convention

ok now and then in terms of preferences, we are going to think, focus on ordinal preferences

we are going to focus on the assumption whereby each agent of each side

let's say has preferencss ordinal preferences they can when called the agent on the other side ok?

so we are going to for the most part focus on strict ranking, another words, agents are preferences such that are you know

he's given agent this not in different at all across two different agents on the other side ok?

we'll talk about the significance of this assumption because that really makes the difference

um preferences are complete and transitive uh strict preferences just like I said

and then rotation that we are going to use uh since it's a syllable um it's tricky I mean it's a lot of mess just to do the struggle thing right?

I mean which is typical use for indicating preferences that   binary relation

we are going to use this just the inequality but to mean basically that this is ordinal preferences for agent k ok?

[25:00]

so or another common way to describe the preference is just to release them, ordinal list of each agent

so here's the preference of agent I M i

so man and his preference is just ordered list of women on the other side

at some point I should have said actually there is the mistake here there should be M i ok?

subscript M I I should be subscript at some point himself sort of pop sup

which means is that anybody below the list is basically dominated by not being match at all ok?

match team himself is being on the side remaining single if you will

so because we are going to focusing on the assignment that matching where which is through individually regional in the sense that I mean

we are not considering the matching where agent is assigned to somebody dominated by remaining single

so typically we'll forget about listing we'll not list agent on the you know was dominated by himself ok?

likewise for each woman. and then we will use term individually rational or acceptable

so woman j agent j is on the other side acceptable to k if j beats k ok?

another words that anybody who is uh comes before himself or herself

we'll say that person is acceptable unacceptable if its list is below himself or herself

so a marriage problem is also tripled ok?

it's descirbed completely by the set of men, set of women and the profile of ordinal preference list, one for each player ok?

and then we considerable later we gonna introduce the term called mechanism and to do so its useful to through think about the set of all possible preference profiles ok?

or that's script p gamma is the set of all possible marriage problem ok?

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so you can vary across the set of men, set of women, and set of profile of preferences ok?

and then when we say mechanism, what do you mean mathematically is a mapping from oh

let me just begin with matching and then mecahnism ok?

so   what we call matching is a mapping from set of agents to the set of agents ok?

such that whenever u is a through mapping is a function that means that m matches the w ok?

so man m is a match to woman w if and only if woman w is match with man m ok?

so this makes perfect sense so feasibility, constraint

here is this basically two side ? constructure for each man and woman either man is the match to somebody some woman or himself ok?

whenever somebody whenever man is match to somebody other than himself he must match to woman that's the two sides restriction essentially so this is called graph

graph is the basically is a couple is the graph, is the set of nos[?29:06] and set of edges

edges are defined for nos [?29:13] across this partition this called by vital pie graph in graph study is not important we'll probably also talk about it later as well

so through the maintaining if you will for the graph tehoretic perspective by vital type of pie graph structure that's all mean by the second constraint ok?

so any such matching, any such mapping is called matching

and then oh so without the last restriction we call it idle roommate assignment problem because there is no real two sign of restriction or we call it one ide of assignment one side of matching

[30:00]

but one side of matching is sometimes used to mean this

the problem where agents are assigned to objects typical resource that location problem this is served

this is problem as old as the economic essential except what's through special about here is that the everything is the creep objects are indivisible and agents are indivisible

so where is the classical economic framework or dealing with the resources and location problem assumed through divisible, divisible good[?30:35] ok?

divisibility assumption is relaxed I mean this is not there through make this problem special anyway

so we are going to stick to the term terminology of roommate assignment because of that because you know one side is assignment one side is matching is often used to mean this or to

mean that which are completely different

uh so for the sake of purpose of this course I am going to use the term roommate assignment problem

although probably we are going to spend any time at all on this we are going to spend a lot of time on this ok?

ok so and then when you say matching mechanism is a mapping from the set of marriage problem to set of all possible feasible or feasible matchings defined for a given marriage problem ok?

so give me a particular marriage problem a mechanism basically determines what matching will happen from that problem ok?

we give the particular specific example of a mechanism and we are going to through introduce next a couple of slides

I think uh one particular mechanism so if you just talk about mechanism you know abstract way it might   be a little bit confusing

but should not be right?

um now so from now on we are going to think about some desirable property of matching

we are not interested in any matching we are interested in some kind of matching

a matching satisfies some property so the first property we are going to talk about the stability ok?

I am going to motivate why we care about this stability later on

let me begin with the definition mathematical definition

so we say matching u is blocked by individual k if the matching u assigns to that individual somebody who is worth than not be matched at all ok? remaining single

so blocked u k prefers being single rather than match according to that mapping ok?

matching uh there is no range somebody thus is acceptable him or her in which case we say a matching is blocked by an individual

when a matching is not blocked by individual

we say individually rational there's no blocking by any so there is no agent the blocks an agent matching   we say the matching is individually rational

math matching suggest partners or acceptable to the agents that are matched to them

it is a little bit more significant description we say matching is blocked by a pair of agents m and w or not currently match according to the u

they are not partners according to u and yet m prefers being match to w rather than his current partner

and that's m's preference w woman w prefers are being match to m instead of somebody suggested by the matching function

if that happens we say a matching is blockd by a pair of agents

we say a matching is stable if it is individually rational meaning that it is not blocked by any agents on each side

and is also not blocked by any pair of agents ok?

so the kind of sort of this is a sense in each that the matching is stable okay, so in other words,

if people are matched are going to match according to this matching function that is stable

it's probably likely to be more bas[?34:53]   in the sense that   there are no you know pair they try to suitable on the mind of matching recommendation by trying to match outside scheme ok ?

[35:00]

simply suggest really set of a point through why we care about stability to degree ok?

as an equilibrium notion which is kind the kind way which thestability was envisaged or introduced by getting shuffling[?35:17] here is some sense of model of marriage market as a positive theory

I mean how who you know come up with any prediction about which man are matched or which men marry which women are as a positive theory

well I mean stability should be through minimal necessary condition right? because if it is violated, either I mean,

nobody I mean if is not individually rational somebody refuses to simply hang out with partners if it um are blocked by a pair of agents is not going to survive right?

that's the sense in each the sort of equilibrium notion right?

this is the equilibrium notion although I mean notice that this is likely cooperative game type of notion we don't we have not yet built a known cooperative extensive form game

and you know apply some through standard equilibrium notion right?

nashy equilibrium, perfect equilibrium. and then somehow this condition pops out ok result of applying standard equilibrium that's not the case at all

this is just the you know condition by self ok?

rules are motivated by you know what happens and what will happen this matching will not survive if this blocking a wide form takes place

but from market design perspective that's not going to be how we want to think about that rather

how we want to think about the ability as a desirable property of matching

because I mean if you think about designing mechanism whereby doctors are matched with hospitals ok?

at least you want to think about ability as a main I mean necessary condition

because its stability is not satisfied that means that maybe incentives on the part of the participant or hospitals or doctors

That means that ,maybe, the incentive in the part of participants

Some participants like hopitals or doctors

They may be sort of strike some idea behind the table, outside the matching system

And that could probably undermine the integrity of the entire matching. OK?

Ah, and then , ah, we are gonna go through.   Look ,A little bit later

But the stability also imply ,ah, nice efficiency notion. OK?

That's a part of the first problem sets

Let me not illiberate all that   not too much

By the way, I haven't really talked about ,ah, problem says . It's evaluation

So, my plan is that when we have two or three problems sets. Ah.. I don't know

If it is useful and important ah, in terms of learning about material by being able to work together

I am willing to let you all together

I would ,ah, from evaluation prospective. Its better for me not to allow you to that

But education is more important

So, if you think that it is important ,ah, if it effects you many cases for, we work together

As a group, then, ah , maybe you should together or ah, submit joint homework or source

But if there are the case. However I will have probably something like ,ah, exam at some point

And , in class exam, I mean

And then, I will app to the this eight classes

I am gonna try to organize some ,um, presentations

I am gonna sign you a reading list ,um, have you present and perhaps by the summary

the paper through you have chosen to present. OK?

Anyway, goint back to the class

This is a desirable feature. In terms of , I mean , just as it says the stability of the mechanism

And it has also nice very pure property. As I said

It has also nice ,ah, pairness property

Ah, because . Why, because it means ,say that one sided school

this side students and the fact that , I mean, so , if we see, find non of the students here.

[40:00]

and so what's the basic notion of fareness?

ah, the most important , I guess, the notion of farenesses and envy-freeness

OK? If system is no matching and fair , nobody envies the assignment of somebody else. OK?

but if schools have priorities, then the freeness is hard to satisfy, difficult to satisfy, not also desireble even from the fareness prospective

Imagine that the schools actually have entrance exams

and students , you know , have different scores, pass scores. OK?

sure, students with very high score will get into a better school , I mean more popular school

Therefore, a student with a lower score will envy the signment of students with a high score. OK?

So, we cannot say the outcome is unfair in the case. OK?

Now, what will be unfair is a situation (of air?) A student has achieved very high score

and yet , could not get into a school which are limited with a lower score student. That will be unfair

So , if you think about ,ah, let's say , students school preference being generated by something like scores

Um, stability means what we call , we will talk about later *more fully?no justified envys* [?41:34] OK

And we justify , um, lets say student ,um, I envy student A and envies is justified

if I have a high priority , lets say , I envies J means that I got, was not able to want it to , but was not able to get into school where J is admitted by

OK? Lets say this *crises*

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I enview J and for that reason , I envy J's assignment . I envy J Ok?

And its furthermore justified.

If ah, S has a higher priority for I than J

Again , imagine that the priority is generated by the past score

That means that I is envy of   J. it would be justify ,if I had a   higher score than J at that school OK?

The fact , I mean , when you say no justify , weeeeee, disrupting situation that's enough all

when we say no justify ,we have a mathing has no justify than B , illiminate justify and B

ah, whenever it is the case that, ah, there is no student,OK?,   who envy , um, no students looses a seat to somebody

who is not as like as the first person by the school . The second guy is *assigned* to

It's a little bit complicated. We will define it more formally later.

But the idea is basically this

No justify envy basically exactly same , one more saying, exactly same as stablility. OK?

Because if you think about woman at school, as a student

That means that , ah, whenever man , let's say just student , ah, envy some other woman Ok?

That means that somebody rather than the score , is assinged to, he likes some other school. OK?

It must be only because the school , W, likes ,let's see, I mean, the fact that it's not blocked by this two agents, means that if this is the case , this inner quality must be reversed. OK?

Ah, whenever M prefers some other school that he could not get into, ok, it must be only because the school perefers somebody admit ,everybody admit, that the school admits

more than this guy. Ok?

The fact that if the matching is stable, it cannot be blocked by any any pair

which means whenever this is the case, the second ineqality must be reversed

And that corresponds to the notion of no justify of envy

So, the task contains the , ah, notion of , ah, finish ,anyway, this is wrong *motivation*

And, I please not to give you wrong *motivation.* anyway,

We will get into there.

Ah, I will speed up a little bit more.

Um, so , here is the big ,ah, punch line

A stable matching is Pareto optimal , OK?, Pareto efficient , optimal , we usually the terms interchangably.

And, It is going to be , ah, um, set of stable matching ,ah, coinside with Core. OK?

[45:00]

So , what is the core?

Ah, Core can be defined in different settings.

But, here, core should be defined in terms of feasible matching. It must be subset of ,ah, feasible matching

core is a set of matchings , OK?

Ah, such that each matching in the core has the property that is no , um, coalition of agnets

Containg man , some subset of man , subset of woman

Ah, booking , get together each other

And do better than the current matching

Another way is that it is not blocked , you can think of the core matching as an extension of stable matching.

So, here , ah, for matching to be stable , must not be blocked any pair of agent

Here, it must not be blocked by any pair of agnents.OK?

What I am saying is that that requiring matching is not to be blocked by a set of agents , bigger that a pair

That's not introduce anymore excution

That's what this statement is saying . OK?

Require more in terms of lubathness[?46:17] to,   blocking

Ah, you don't gain anything . OK?

This is true .enough

In another way, additional coalition doesn't come into play , which is kind of very easy-to-undertand

Because everything is pairwise. OK?

If there is a blocking coalition , you should be always able to find a man or a woman who should do better

therefore you should be able to firnd the blocking pair

Whenever there is a blocking coalition bigger than a pair of man and woman

who should block the original matching

Ok. That's the argument. I am asking you to think about it carefully but nothing much more than that

Actually, it is ,ah, what you need to do for the problem set

just nice , right? The study let this correpondence imply Pareto optimality. OK?

You are also more careful … ah, everything, every Pareto optimaility matching is also stable.

I dare say that. What I said is that every stable matching is Pareto optima . So , one direction

But the set of Pareto.. Set of stable matchings coincide with set of core matching

That means set of nice notion . That's what I am trying to say

ah, OK. The question we ask ,first of all , is that ,ah, given the structure of the problem known as marrige problem , thus, there exists the stable matching all the time.

Can you expect to give me any marrige problem

In terms of stripers at a man and women and their associated

Profile preferences.   Can you be sure that stable matching exists for all possible marrige problem that you can imagine?

Answer would be Yes. Ah, you shouldn't take the , you should be impressed , yes, supposed to be impressed with the existance

Why ? Because , should not be things like stability can not be taken for granted

Ok? I can give you a simple example

which cannot be two-sided matching . Of courese , because of the contradict but I said you

So, think about the roommate assignment, roommate matching problem

That are , um , let's say four agents, A,B,C and D. OK?

A's preference is like this , like B ,C and D

B's preferece is like ah, C , A , and D

C's preference is like , ah , A , B and D. OK?

So, one thing you should be careful about this example, ah, is that this is *the list like person sort of*

Most dislike person, OK?

So, again, one to one matching , but ,we , without intersection of the two sideness. OK?

So, anybody can do match with anybody , so , Ok?

imagine , ah,   give me a feasible matching here. Ah, what , you should come up with two pairs it's essential , right?

Ah, for any feasible matching , however, so, um, what is true is that I am gonna prove that the distablility(or this stability) will not work in this case

There is no stable , ah , roommate assignment in this problem.

Why? Because , imagine ,ah, in any feasible matching . OK? Somebody must be , ah, match to D . Ok?

[50:00]

Let's say that the person is B

So, B is matched with D

And then, A must be matched with C, OK?

Now, So, B guy is a lucky guy,   and he looks around and who is the most popular?

To whom is the most popular guy?

OK. It is A.   OK?

Then, he goes to A and suggest matching each other . OK?

He , of course, prefers to being matched to A rather than matched to D

And A likes that idea of lucky

why? Because he is the first guy . Right?

So, you can do this , ah , argument , no matter who is matched to D. OK?

Anybody here, the way the preference is sturecured is such that ,ah , anybody is the first guy for somebody rather than D

Right? So,   I didn't even describe this preference. Because that's really matter.

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But. To be complete,   You might wanna, ah,   through this prefereces. You know

So, there is no stable matching here. OK?

And when we say there is no stable matching , what we really need is that we cannot guarantee existance of stable matching for all marrige problems OK?

So, you have to be careful , I mean , specially if you are not , ah, you should , to , style of statements that are common in economic general

Ah, open, I mean, we have to be careful about the qualifies like for all or for any , you know, there is existance thing like that. Right?

So, when we say stable matching exists , what it mean is exist for all marrige problem. OK?

And the argument is very nice

nice in the sense is constructed

So, we are gonna actually ,ah, develop an algorithm and if we run through these algorithm, at the end of the learning , through the algorithm , we will have found the matching that is stable. OK?

So, it's an invention of Gale [?and Shapldy 52:32]

Ah, David gale is famous , applied mathmathician and also with a lot of significant contributions to economics

[?Roe Shapldy 52:33] Is , of course, very well-known for a lot of problem, ah, a lot of contributions. [?Shapldy 52:40] values and um, ah, assignment markent, assignment games, you know , a lot of important contributions. OK?

So, there are two different variants of the algorithm that they propose

And, what we call , men proposing   DA, deffered acceptance

So , DA represets deffered acceptance algorithm OK?

And, there is another counterpart . Ah, it's a women proposing deffered acceptance

I am gonna explain how these two differ. OK

I am gonna use the , um, acronym and PDA open that meas ah, men proposing deffered acceptance. OK?

[?Sivenie Scott 53:26]'s assumption of strict prefereces , like I said before, but , ah, if the prefereces does not complete this strict , ah , To learn this algorithm you need to need chik rankings[?53:36]

Ah, so, you will need to do something like , you have to break the ties. OK?

Maybe , by using something like a lottery.

But there is how it goes . OK?

Some of multi stage , final stage ,ah , algorithm , Since page no.1

Each man proposes to his most prefered . So we need , I mean , given that it is a problem , ordinal preferences are well-defined. OK?

So, we start , for each man , we start with the first person , first moment on this list

There is no first woman if he starts with himself , then he dosen't propose to anybody. OK?

And, if he was his first choice , as long as that the first choice is acceptable. OK?

So, nobody proposes to unacceptable person in this problem

which guarantee is that as long as , you know, somehow , you are matched . According to the way you propose in a certain way I am gonna explain later

individual nationality will be guaranteed So you can see why , I mean , what different , how the algorithm is set up.

To guarantee , not only I mean stability and , you know, I mean that , individual   nationality

Each man proposes to his most prefered . So we need , I mean , given that it is a problem , ordinal preferences are well-defined. OK?

Each man proposes to this first choice as long as that is acceptable

And then, each woman , then, ah , look at who propoese to her , him , her . Rihgt?

And, if nobody proposes to her , then , there is nothing for her to do. OK?

[55:00]

If somebody proposes to her , then, the first thing to check is whether that person is acceptable to her . OK?

And , she accepts anynody who is never unacceptable.OK?

If there are mutiple , if there is only proposal from unacceptable mate , acceptable man

Then, she accepts that man , but only tentatively , I will point

So, that's also with a mutiple proposal , mutiple proposals from multiple men

Then , ah, she would choose among those proposals , the best, according to her own preference list

As long as the person is acceptable

And reject all others. OK?

So, rejection is final OK?

the algorithm must proceed in the monotonic way.

which means that something must be final.

rejection is final.

acceptance as you see is not going to be final.

rejection is final in the sense that if I am rejected, if I am man rejected by my first choice then I can never reapply to the same person okay?

so then I have to move down my preference list.

so in general lets say this step number 2 okay?

now people who propose have been accepted they have nothing to do.

all the man are on the acting people applicants are those who are rejected by their first choice.

those who are rejected by first choice, are apply to the second best choice as long as the second best woman are according to his preference list is still acceptable. okay?

again it works in the same way as before expect one important force

if I am were woman, again I do same thing.

I may have has somebody that I had accetped in the first round.

I still have that, I keep that proposal

if I have nothing then, that's fine.

it 's like first step.

in the any? regardless, that may be additional proposal coming through coming in.

and again then I have received everything on the proposal when they could be zero, or one or multiple. okay?

and then I have one proposal that I held from the previous step.

I look at all of them.

and then I pick the best one

again I mean she doesn't consider unacceptable proposal, among the acceptable proposals for the current step as well as the one she held, from the previous step.

she picks the best according to her preference ordering which is well defined.

it goes on like and that rejected everybody else,

again rejection is the final.

The those who are rejected by their second best choice must be applied to their third best choice as long as that is acceptable.

it goes on like that until there is no more rejection.

and to first thing to note here is the algorithm is going to be finite, because number of players are finite.

why because rejection is final in each step.

so this algorithm terminates in final round.

so it's well defined

one thing that is also important to notice is that as the algorithm proceeds, okay?

you can usually see why the end outcome, which is matching I didn t say which is best important these things, in the last step when no proposal is rejected,

then the algorithm is end as I said.

and the matching that   there tentatively, would be the final matching.

so in the last step, some women would hold their proposals, one proposal for each okay/ and each women assign to that man.

so algorithm produce some matching, feasible matching. okay?

and as you usually see, it is a feasible matching that is no man is matching to another man different from himself, okay? likewise for woman.

[60:00]

you usually see the individual nationalities guaranteed because no man ever are proposed to unacceptable woman. okay?

so therefore, the end outcome in terms of matching is not going to assign woman, but is only acceptable to each man

likewise no woman ever accepts a proposal from unacceptable man.

which means that it is going to individual nationalities rational for man as well

as you can see as the algorithm proceeds, woman gets better and better because you have an option,

you know man I have proposal I can do no worse than the proposal

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man can not broud[?1:00:41] you know his *offer*,

it's my choice is woman to kick that option alive

I will switch to somebody else only when I like that person you proposal is better

so as the algorism proceeds, the woman can only get better. okay?

so opportunity set growing in some sense.

for the woman side, opportunity said, dwindling as it proceeds for man side, which means suggest that this is not a good deal for man

but actually this is a better than the other counterpart which is woman proposing deffered the acceptance algorism.

so why do we call deffered acceptance as you noticed the rejection it instead it is the final but the acceptance is deffered, deffered until you ask last step.

let me just do this proof quickly, and take a five minute break or three minute break.

stable[?1:1:51] matching exists for every marriage problem.
and its constructive meaning that the deffered acceptance algorithm of either kind, okay, produces matching always which well defined meaning that it always produce some matching.

and then that matching is going to be stable. okay?

so the first of all, this is the existence spot, final miss means that the algorithm can never go in definite.

because there will be the case, rejection is also tentative.

but you know, we have so many women to propose to, because you get rejected , you are not turn around and can re-propose, propose again to the person who rejected you

if that would be case, you can go on forever, but that's not the case.

stability parts, there are 2 components, I proof the matching that comes out of it.

[?1:03:01] is individually rational, and also cannot be blocked by any pair.

so individual rationality is already shown, man never proposed to anybody unacceptable, woman never accept anybody who is unacceptable.

now so only need to show is that there is no blocking pair.

and that's not also very hard

because lets say, whenever, supposed that, the exist of man and woman, the blocked such that man likes that woman more than his assignment given by this algorithm at the end the of step.

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obviously they are not currently matched each other.

suppose this one of case, when can we *dedure?* from the way in which this algorithm work?

we know that the man the order in which the man approach to woman is according to his preference.

the fact that he likes woman, better than his partner arranged by this matching mechanism means that this man must have approached in this mechanism to that woman before he did to her.

the only reason that you know, he end up getting match with this woman

because, he had approached to her, and then was rejected by that woman.

now we can not say that we only know that the man , this man is a of course,

supposed that their individual rational, if that man is unacceptable, than of course, that's the easiest story,

because that this cannot be a blocking pair.

suppose, therefore, this man is acceptable to this woman, that s the only way that this can possibly, be a blocking pair.

in that case the reason that man is rejected because she has a better proposal at that time.

or at that time, sometime it could be that he was initially accepted tentatively but then later kicked up when she got the better proposal, doesn't matter.

so she was eventually at some point, rejected him,

when revise that, she does that in favor of some other man, nobody, no woman reject an acceptable man, without having a better alternative, according to way that algorithm works.

so that means that, there exist some men, such that, woman likes that man, more than him.

now remember that as this algorithm proceeds, woman can only get better, meaning that even that man, may not be her eventual partner.

if it's not, it is only because that eventual partner even dominate this guy as well.

so what we can conclude, is for this woman, this must be the case,

so riquil [?1:7:12]quality it means that I mean since we should make preference for everybody.

riquil[?1:7:17] equality means that either this person is different from that person as treated dominates him for this woman or the same person which is both possible.

either way however, what we know is that whenever if there is a pair unmatched pair, such that man prefer strictly that woman over this partner.

it must be the case that woman actually likes her partner more than that man, thus this can not be a blocking pair.

so we have shown there is no a blocking pair.

one way to show suppose that the blocking pair, and then we show that you know, this is the case/

therefore we have a contradiction

or we can simply say if this would be the case,

this must be always the case,

therefore, we can not have a blocking pair.

let me stop for three minutes